Combinatorial Complexity

by Kate Jones, DSPE

Ah, puzzles! Here is my recently designed new tiling set, Tri-Chex,[1] that encompasses math and art. A few illustrations will give a look inside the subject.

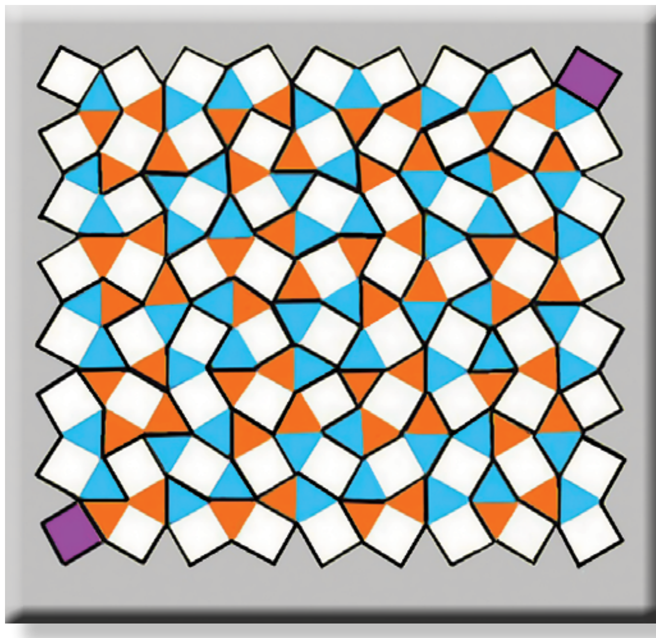

Figure 1: Tri-Chex with its 53 pieces joined, non-matched colors.

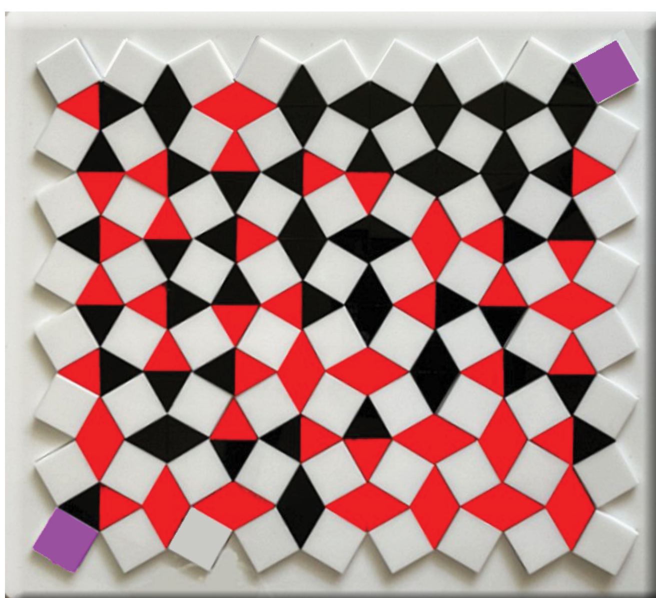

Figure 2: Tri-Chex with all 53 pieces joined, 28 pairs of matched colors.
Not the maximum; the search goes on for the theoretical 32.

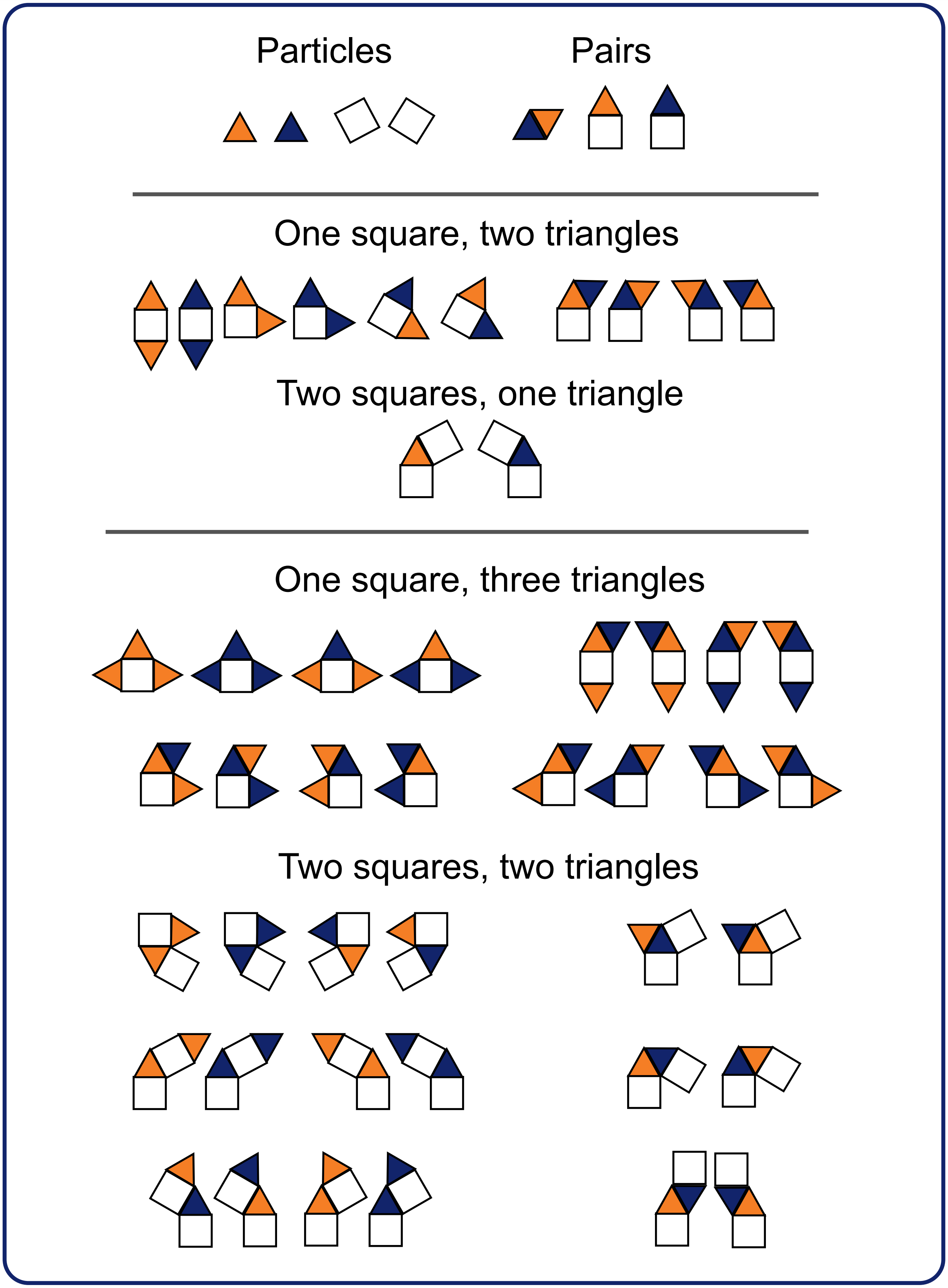

Figure 3: The 53 components of the Tri-Chex set.

The non-matching solution (Fig. 1) may be one of the most difficult I've ever tackled. Finding the solution was something of an "It's a perfect world" moment. That the tiles contain from one to four component polygons, all different and including all possible shapes and adjacencies, is an aspect of "combinatorics."[2] If there were only single polygons, it would be a breeze to assemble them into the defined color mosaic. Having all different shapes and sizes of pieces (Fig. 3)—that need to nestle together and still observe the color conditions—presents an outlandish challenge.

Combinatorics is an unaccustomed word, familiar mostly to professional mathematicians. Philosophers can also employ it when discussing all the permutations and combinations of the world and of the galaxy (and all the universes, while we're at it) and the processes of evolution, devolution, time and space and gravity and matter and energy and consciousness and all that we don't know yet. We ponder cause and effect, yet every moment the combinations of interacting elements are unpredictable. Everything does tend to line up in systems and patterns, at least temporarily, which brings us back to the puzzle under discussion. Let's see if on this tiny scale we can exercise more control than over the unfathomable universe.

This tessellation is related to a quilt pattern formed of only squares and equilateral triangles, with a unique characteristic—the three-color array contains a type of checkerboarding with all three colors:

- The squares tilt left and right to join corners in an infinite field of "periodic" repetition of their 2x2 quadrants.
- The triangles of one color form four-armed stars touching their own color only at tips of corners in a type of checkerboard made of rows of pointy squares.
- The triangles of the other color also form four-pointed stars, hovering between the other stars like a jagged checkerboard.

Here are a few other peculiarities of this tiling effect:

- Each square is surrounded by four triangles hugging its sides. The exceptions are squares on the border of the array, but keep in mind that it is an infinite plane. We've merely carved out this section, so outside edges are exposed.
- Each triangle touches two squares and one triangle of the opposite color.
- Every triangle joins one other triangle, forming a diamond or, more precisely, a rhombus. In the canonical solution (Fig. 1), every pair is made of two colors.
- Convex shapes are quite rare. Two single pieces—the square and the triangle—are each convex. Joining two triangles or a triangle and a square also makes convex shapes, with, respectively, four and five outside edges. The only way three parts can be convex is a square with two triangles, one each on its opposite sides; that shape has six edges. Two squares plus three triangles can also become a convex form, surprisingly with seven facets.
- The largest possible convex figure is built of 12 tiles: a stretched decagon containing four squares and eight triangles. No matter how many more tiles you connect, up to infinity, you will not be able to get a larger convex group. Strange, isn't it?
- That 12-part stretched decagon is an interesting silhouette: look closely at the full tiling in the 8x9 rectangle (Fig. 1) and you'll see it form a sea of these "ovals" or "eggs." They go up and down as well as sideways, and they overlap slightly with their neighbors in both directions. You can build infinite chains of these eggs on the cosmic plane. A tiny sample is the letter "I" in the TELICOM solution (Fig. 4).
- The three-color formula for this tiling was chosen, from among millions of possibilities, for its strong regularity and consistent symmetry. Identifying all the possible shapes

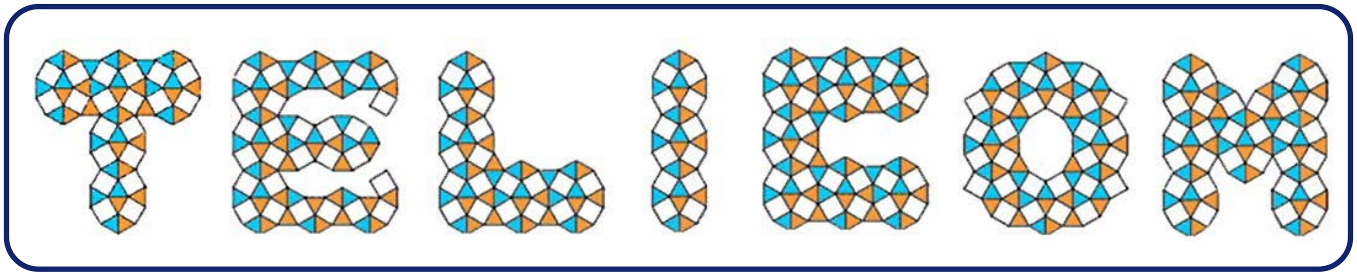

Figure 4: Letters of the alphabet built out of Tri-Chex “eggs.”

of combining 1, 2, 3, and 4 unit cells and then placing them upon this tessellation brought out the 53 distinct tiles that make up the set. While it could be argued that having two single squares creates one duplicate, the second square is tilted the other way and is needed for parity within its dedicated 8x9 rectangle.

Here’s a bit of formal lingo borrowed from the professionals:

- Mathematicians going back to Johannes Kepler[3] in the 17th century have studied many 2D and 3D geometric arrangements, and this particular one with squares and triangles is also called, after Kepler, a “snub square tiling,” a semiregular tessellation of a two-dimensional plane.[4] It’s not to be snubbed!
- All participating polygons match by equal lengths of sides. The triangles are all *congruent*, and the squares are all *congruent*. Each *vertex* is a meeting point of three triangles and two squares. Math shorthand shows that as 3.3.4.3.4.
- John Horton Conway,[5] the 20th century’s most legendary mathematician (I knew him personally), called it a “snub quadrille,” constructed by a snub operation applied to a square tiling (quadrille).

Designers and researchers are allowed to invent the words to go with their subjects. Hence my name for the set is Tri-Chex (a triple-checkered pattern). And, instead of “polysnubs,” I call them “polyfans” (after the shape of a square with two triangles on adjacent sides). Aren’t your fingers just itching to try arranging them your way?

NOTES. . . . .

1. “Tri-Chex,” Kadon Enterprises, Inc., http://www.gamepuzzles.com/edgmtch4.htm#TCX.

2. “Combinatorics,” Stanford Mathematics, School of Humanities and Sciences, https://mathematics.stanford.edu/research/combinatorics.

3. Robert S. Westman, “Johannes Kepler, German Astronomer,” *Britannica*, https://www.britannica.com/biography/Johannes-Kepler.

4. “Snub Cube,” Wolfram MathWorld, https://mathworld.wolfram.com/SnubCube.html.

5. Catherine Zandonella, “Mathematician John Horton Conway, a ‘Magical Genius’ Known for Inventing the ‘Game of Life,’ dies at age 82,” News, Princeton University (April 14, 2020), https://www.princeton.edu/news/2020/04/14/mathematician-john-horton-conway-magical-genius-known-inventing-game-life-dies-age. **Ω**