Combinatorial Complexity

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Abstract

This document in poetic form is an exploration of the tessellations and symmetries of polyform set *p4g*, one of the 17 wallpaper groups.

Tri-Chex

Ah, puzzles! How the thoughts of mathematics roam
To find the pattern's law… and catch it in a poem.
What have we here, of squares and triangles arrayed,
A tessellation or a quilt, precisely laid
In periodic spacings, with each vertex bound
By polygons of 3.3.4.3.4 around. *(Figure 1)*

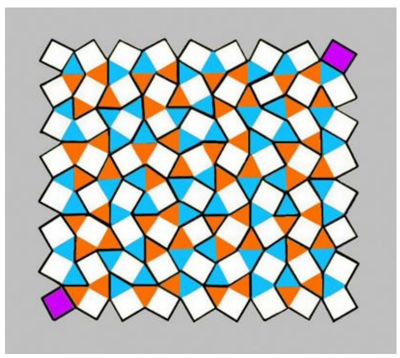

Figure 1: *Tri-Chex color-separated solution.*

Behold how tilted squares at corner points attach,
With triangles well sized on every edge to match.
The triangles with non-matched colors paired appear
As rhombi forming checkered stars. Detect them here.
Two triangles that face across a square reflect
The self-same color, every pair that you inspect.

Three colors shared on 53 distinctive tiles, *[Figure 2]*
Each one unique, make 1-2-3-4 size of piles.
The combinations are complete and intricate,
As convoluted as plain polygons can get.
To win assembly so the right mosaics nestle,
Non-matched, reveals to be a mighty mental wrestle…
Outlandish, even, if without your robot's aid
By human skill alone—be warned—this feat be made.

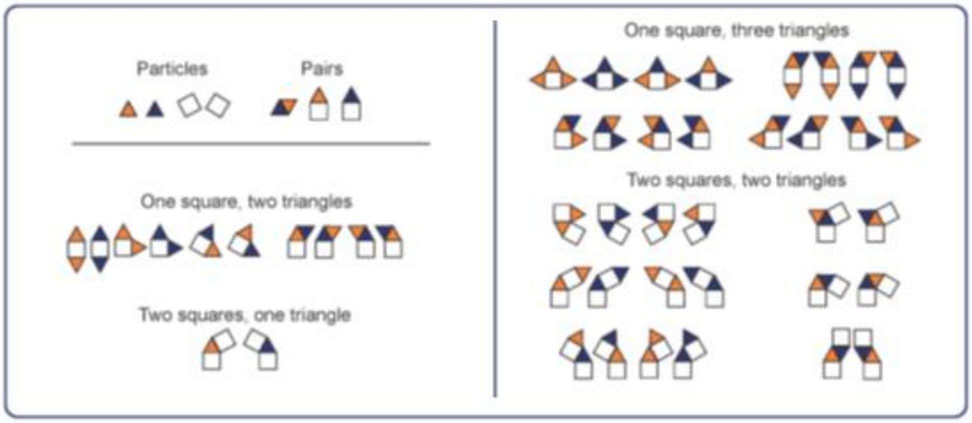

Figure 2: *The 53 unique Tri-Chex tiles.*

A touch of history—the 1600s glowed
With wit and wisdom, as Johannes Kepler showed.
He studied 2D, 3D tessellations grand
And added language to spread math throughout the land.
Let squares and triangles a semiregular plane
With pleasing symmetry as "snub-square" tiles entrain.
Strange definition, this, whence such a term was dubbed;
Still, Kepler's fame persists, his choice not ever snubbed.
Three centuries later, a most legendary mind—
The wiz, John Horton Conway, "snub quadrilles" opined.
Lest "polysnubs" invade our Tri-Chex naming scheme,
Let us rename them "polyfans", for so they seem. *[Figure 3]*

Figure 3: *The "fans"—the smallest symmetrical concave Tri-Chex tiles.*

To be contrarian for once, adjoin same hues, *[Figure 4]*
When building 32 one-color rhombs you choose.
Here is a start, four short of maximum success.
Send us your win, a prize arrives at your address.
Now study well the structure of yon plate of tiles
Wherein recurring decagons form rows of files,

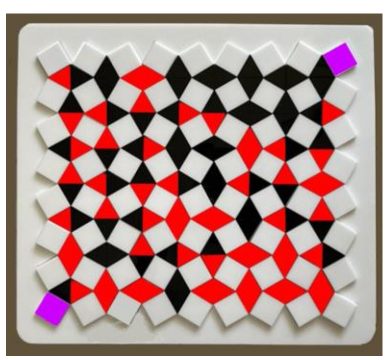

Figure 4: *The adjoining 28 matching colors; not a maximum.*

With vertical and horizontal overlaps
In infinite expanse that has no gaps.
Such tessellated periodic planes appear
In quilts and art, wallpaper patterns far and near.
Perceive thee how, like fields of oval eggs they cluster
When eight triangles and four squares ye neatly muster.
With eggs alone create and build such splendid art *[Figure 5]*
That all who see will want therein to take their part.

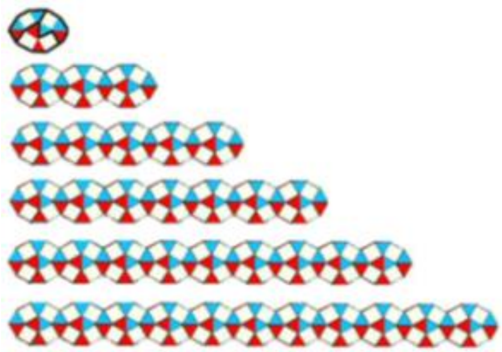

Figure 5: Forming chains of "eggs" or convex decagons.

To improvise a tasty literary omelet, *[Figure 6]*
With eggs and cleverness design an alphabet:
If ye have fallen under symmetry's sweet spell,
The other letters will yield to your search as well.

Figure 6: Letters made within 3x3 arrangement of eggs.

Another famous group of shapes for you to solve *[Figure 7]*
Are the pentominoes, all twelve the eggs involve:

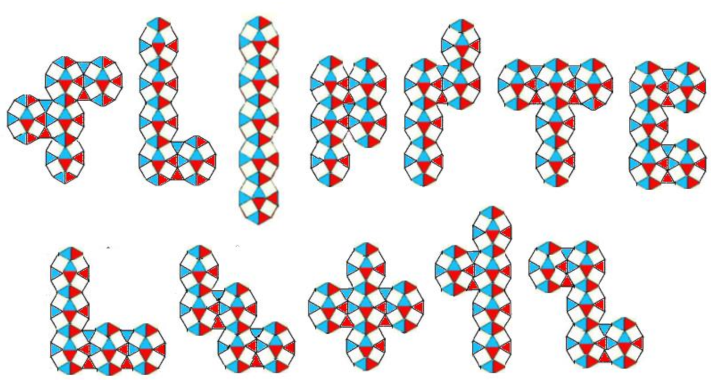

Figure 7: The 12 pentominoes formed with 5 eggs each.

Now here's another curious and surprising fact:
That egg's the largest convex shape that can be packed!

It will not grow beyond that decagon's small start,
Not even if you took the Universe apart.
Conversely, there's no end to concave figure tricks
Whose clever symmetries in every way will mix. *[Figure 8]*
From small to medium to large, what size you build,
With one to 53 tiles, treasure what you've filled.

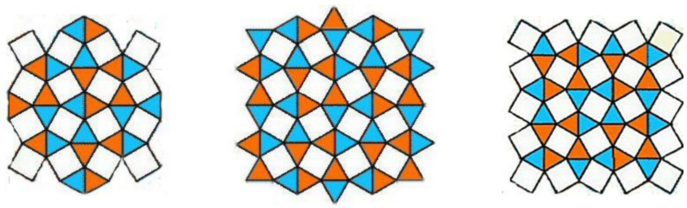

Figure 8: A few samples of concave figures.

Shall we another theme try in the Tri-Chex tray?
Upon their colors, with no contact let them stay,
As many size-4 islands as can fill the floor— *[Figure 9]*
Eighteen we've found. Can you squeeze in at least one more?

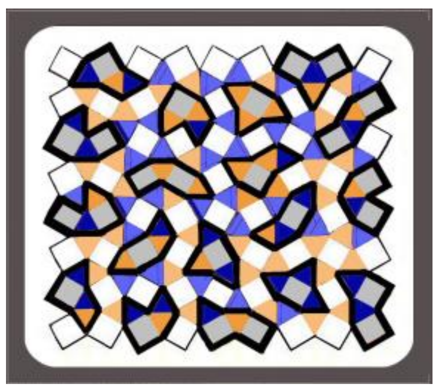

Figure 9: Islands—tiles match the floor colors without touching each other.

Conclusion

Such recreational math is more than just a toy.
It's art and science—finding answers brings you joy.
The human mind vast fields of understanding craves,
The world with symmetry and ordered beauty paves.
From singularity infinity it chases,
Hence we LOVE EVOLution and its mirrored faces,
And every change begets diversified delight,
That's how we build a consciousness, no end in sight.
Cause and effect, yin yang, a megathought above:
The Universe's timeless symmetry is… love.