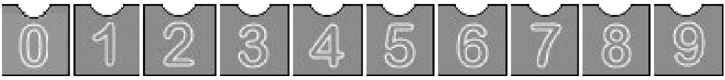
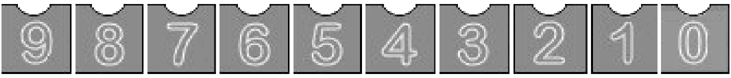
For 1 to 4 players
Ages 10 to adult



Joe Marasco's

RunnuRound™

The backwards and forwards game of probabilities and smart guesses



Game Rules
Solitaire Quests

Contents:

- 1 game tray (deluxe) or felt wrap
- 40 number tiles (10 each in 4 colors)
- 4 "wild" tiles (1 each in 4 colors)
- 12 disks (4 each with 1, 2 and 4 dots)
- 4 "queue" disks (numbered 1, 2, 3, 4)
- 4 bamboo screens
- This rule book

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Introduction

by Joe Marasco

RunnuRound is a fun game that can be played by strategy lovers of all ages. It helps develop the ability to see patterns and, in advanced variants, to make predictions about outcomes based on partial information. It also shows in tangible form some fascinating problems in the mathematics of probability.

Concept of the game

Have you ever considered what happens when you take ten numbered objects and place them in a row?

Try it now. Take one color's worth of tiles numbered zero through nine, and place them in no particular order side by side in a row. What do you see?

Most people will almost immediately detect consecutive numbers that are adjacent. For example, children who have learned to count will exclaim, "There's a 6 next to a 5!"

This reaction becomes reflexive as we get older, the result of subtle programming deep in our brains. We *see* the pattern without consciously *looking* for it.

Similarly, we can quickly identify when such a pattern is not present, and we call such a layout "random"—an informal way of saying there's no pattern we recognize.

The simple act of laying out ten tiles in a row leads to some interesting mathematics. For example, there are exactly 3,628,800 different ways to do this.

This is a surprisingly large number, resulting from the idea that you can put any of 10 tiles in the left-most position, then any of the remaining 9 in the second position, and so on. This leads to 10 x 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1 different arrangements.

Mathematicians call this number *ten factorial* and write it as 10!, so 10! = 10 x 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1 = 3,628,800.

Runs of 3

Now let's consider a slightly more complicated pattern. Instead of looking for two numbers that are consecutive and adjacent, let's consider "runs" of three consecutive adjacent numbers. That is, the pattern we are looking for is something like a 6, 7, and 8 all next to each other in that order. We'll denote such a run as {6 7 8}.

Note that {6 8 7} is not a run, because although there are three numbers that could be consecutive, they are not in the right order: 8 does not come between 6 and 7.

On the other hand, {8 7 6} *is* a run; instead of ascending left-to-right, the numbers now go in descending order. But because they are adjacent and in the right order, they are still a run. However, regardless of whether the run ascends or descends, we count it only once: No double counting allowed!

Stop for a moment and consider: How often do you think runs of 3 occur?

How should we count runs that are longer than 3? In this game, we consider that a run of 4—for example, 6 7 8 9—is really 2 runs of 3: {6 7 8} and {7 8 9}. A run of 5 would be counted as 3 runs of 3, and so on.

Wrap-around numbers

Now let's make it even more interesting. Instead of thinking of the numbers 0 through 9 as a train, with 0 as the locomotive and 9 as the caboose, imagine them going in a circle: We attach the front of the locomotive to the rear of the caboose on a circular track. What happens then?

We have some new runs: {8 9 0} is a run, and so are {9 0 1} and {0 1 2}; we say that the numbers *wrap around*.

But, wait a minute. That means that {0 9 8}, {1 0 9}, and {2 1 0} are also runs, because runs can either ascend or descend.

Wrap-around rows

And we can make it even more exciting. We can decide that we are going to let the row itself wrap around. That is, the right-most tile in the row—the locomotive—gets attached to the left-most tile—the caboose—in the row. So now, for example, looking at all ten tiles, we could have 8 9 X X X X X X X 7, and because the row wraps around, we now have the run {7 8 9}.

The combination of number wrap-around and row wrap-around can lead to some interesting possibilities. For example, how many runs of 3 can you see in the sequence 1 2 8 7 6 3 5 4 9 0?

There are three: {8 7 6} in the third, fourth and fifth positions; and 9, 0, 1, 2, a run of length 4 wrapping around the ends, yields another two that overlap: {9 0 1} and {0 1 2}. Clearly, overlapping runs of 3 are possible, but they occur less frequently than single runs. Can you think why?

The name of the game

The double wrap-around feature for runs of 3 is what gives the game the name RunnuRound. Both Run and nuR are “runs” of three letters, with Run and nuR evoking the idea that the runs can go forwards or backwards; they are mirror images of each other. And RunnuRound nicely has 10 letters, the number of tiles in a row.

Probabilities

So now a puzzling question arises: If there are 3,628,800 ways to arrange the ten tiles in a row, how many of these arrangements will lead to exactly one run of 3, as we have defined runs with all the various possibilities: ascending, descending, with number wrap-around, with row wrap-around, and with more than one run embedded in longer sequences?

The math is not so easy. Give it a try! We'll tell you the answer so you can check your calculations: There are 611,200 ways to get exactly one run of three in a layout of ten numbers. That means that a single run will come up 611,200 times out of 3,628,800 or about 16.8% of the time.

If you didn't attempt a computation, what did your intuition tell you?

To make our game more interesting, we lay out *four* rows—one of each color—at a time. This gives players more things to look at, and more runs to identify. Remember that runs are limited to their own horizontal row (color). Can you predict how often there will be at least one run of three when we play four rows at a time?

In order to do this, you have to know that multiple runs of 3 occur about 3.2% of the time, so that the overall probability of success in any row is close to 20%.

Did you get a success rate of about 59%? *Hint:* The probability of failing is the probability of failing in all four rows, which is 0.8 x 0.8 x 0.8 x 0.8.

Still more…

Another approach to calculating the probabilities is made possible by today's computers. One can simulate the shuffling of the tiles in a program, and then look at the results for a large number of shuffles. This approach is often called the Monte Carlo method, and computers can do these tasks very rapidly. We generated 10,000 single-color simulations, the equivalent of 2,500 games, and the results were within 1% of our previous calculations.

We hope that your appetite has been whetted, and that you will continue to explore the wonderful world of combinatorics and probabilities.

Now, on to the detailed rules of the game. Happy hunting for spotting all the runs of 3 you can!

***Sidebar*:** For geeks of all ages, we point out that computer scientists call our wrapped-around row a *circular buffer*, and that there is an extensive body of technical literature on the subject.

In investigating the probabilities associated with this game, we explored mathematical territory that had not previously been trodden upon and found new results worthy of inclusion in that literature. So our simple game has deep mathematical roots.

The results of the research for this game can be found in the *Online Encyclopedia of Integer Sequences*,

http://oeis.org/search?
q=marasco&sort=&language=&go=Search

It is another example of a curious fact: Sometimes situations that appear to be simple have explanations that are complex. Nature takes simple elements, combines them in complex ways, and arrives at results that ironically appear to be simple.

-- Joe Marasco

How to play RunnuRound

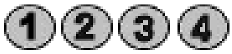
for 2 to 4 players

Random Scramble version

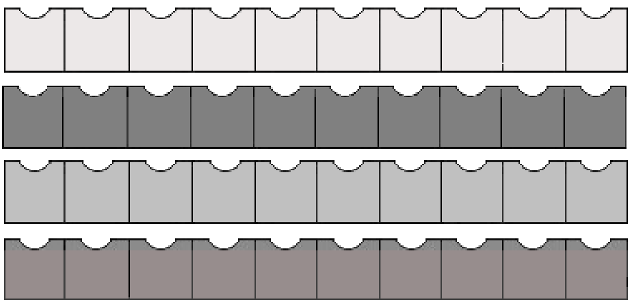
Start:

Each player gets one screen and a set of three prediction disks—1, 2 and 4 dots.

Place the four queue disks (numbered 1 through 4) off to the side within reach of all players. These will show in what order the players enter their predictions.



Place the 40 number tiles face down on a smooth play surface and shuffle them around, mixing well, with all players getting their hands into it. Then arrange them in rows by color, all the arcs oriented in the same direction:



Once lined up, everyone helps to flip them over. As an option, each player by turns flips over 5 or 10 at a time in any one row. It's easiest to flip them top to bottom, using the arcs for leverage.

Play:

As soon as all tiles are visible, or even before then, players try to see how many runs they can find or, more to the point, how many runs they can *predict* will be found.

As players decide on a number, they grab their prediction disks, using combinations of 1, 2 and 4 to mark the predicted number, 1 to 7, and hold that number concealed in their hand while hiding the unused disk(s) behind or under their screen.

In the order in which players make their choices, they take one of the queue disks, with first player getting disk 1, second player disk 2, and so forth.

When all players have finished making their predictions or counts, they compare their results and total their scores.

Scoring:

If there are disparities between player counts, verify where all the runs are, or if there were zero runs. Any player with an error gets no score and no bonus. For correct results, players receive one point for each run, and bonus points for their place in the queue:

1st player:	4 points
2nd player:	3 points
3rd player:	2 points
4th player:	1 point

A player with incorrect results drops out of the queue and the other players move up.

When there are zero runs, all players who correctly called it score one point, plus their bonus in the queue.

Tough-player option: For errors, deduct one point for each run miscounted.

Goal:

To be the first player to reach 10 points by making the closest predictions and in the shortest time. The queue bonuses reward those who can make rapid decisions under the pressure of uncertainty and partial information.

In case of ties…

Play as many rounds as necessary for one player to reach 10 points. In case of ties at 10 points or more, play additional rounds until the tie is broken. If players choose to use the tough option of deducting points for errors, the endgame becomes very suspenseful as some players in the lead may end up going backwards.

“Wild” tile variation

Play according to the rules for the Random Scramble version, with these additions:

In addition to the basic 40 number tiles, mix in the four wild (#) tiles. When the players arrange the four rows of 10 tiles, all still face down, the eleventh tile for each color will be set aside and left face-down.

After the 4 rows of numbers are revealed, players may assign any number (0 to 9) to the wild tile, even differing numbers per player as they visualize a run.

Including the wild tiles may well enable a few more runs, thus your predictions will need sharper intuition and calculation. Scoring is as outlined on page 9.

The “Elevator” wrap

Up till now we have counted runs contained within the same row only. In this advanced variation**,** a partial run from the end of one row can wrap around and connect to the other end of any other row as well, skipping up and down between levels.

Look at the first layout below, where a run that starts with 6, 7 on the first line can drop down two floors to meet up with the 8 at the other end to form one run:

X X X X X X X X **6** **7**
X X X X X X X X X X
8 X X X X X X X X X
X X X X X X X X X X

In the next example, we can count the 6, 7 on the fourth line as part of two runs—riding up to each 8 in turn:

X X X X X X X X X X
8 X X X X X X X X X
8 X X X X X X X X X
X X X X X X X X **6** **7**

And here is an example of four runs, where each 6, 7 can wrap to each 8:

X X X X X X X X **6** **7**
8 X X X X X X X X X
8 X X X X X X X X X
X X X X X X X X **6** **7**

Clearly, this variation will require much more thorough searching, all the while under time pressure to make quick predictions so as to get those valuable queue bonuses without wiping out through an error. Play this variation when you need an adrenaline rush. It is best played *without* using the wild tiles.

Vertical wrap variation

Escalating the complexity of RunnuRound, players can choose to include runs that go straight up and down in any column and wrap around their tops and bottoms. Count these runs in addition to those you find in any horizontal row. Double the suspense and the risk. It is best played without using the wild tiles. The same number can be part of more than one run. See the 4 in the {6, 5, 4} run and the vertical {4, 3, 2}. Another run wraps in the seventh column, {5, 6, 7}.

X X X X X X **6** X X X
X **6** **5** **4** X X **7** X X X
X X X **3** X X X X X X
X X X **2** X X **5** X X X

Draw Bridge version

RunnuRound with strategy and tactics

Start: Players get one bamboo screen each and stand it up as a wall to block other players' view of their tiles.

Place the 40 number tiles face down on a smooth play surface and shuffle them around, with all players participating.

Decide order of play. Taking turns, players draw 4 tiles at a time, one of each color, and hide them face-up behind their screen.

For a 3-player game, the final draw is one tile each. Place the remaining tile as the start of a row of that color.

Partnership option: With 4 players, facing pairs of players may collaborate as silent partners—no verbal consultation.

Play:

Once all the tiles have been drawn, players take turns placing *two tiles at a time* (the same or different colors) from their "hand" into a 4x10 array, each color in its own row.

The first tile of any color starts at either end of its row. Add further tiles of that color immediately next to a tile already placed in that row or starting at the other end.

On their turn, players score one point for any run of 3 they form. Multiple runs may occur and are claimed at the time of play. Keep a tally of each player's score.

Players' strategy will be whether to enable or to block, since a subsequent player may extend a previously made run to 4 and get credit for it again plus the new one added. Another player could then add another tile and make it a run of 5, scoring 3 points.

Things get even more intense as play gets close to filling a row, and especially when there are only one or two turns left. The last tile, if at one end, may produce a wrap-around to the first tile in that row. An interior tile may have been saved by a savvy player for a multiple run.

Note also that when they get near the end, players can deduce which tiles of which color are still outstanding, based on knowing which ones they have.

Play four rounds (three for 3 players) so each player has a turn at playing first.

Scoring:

Keep a cumulative total of points scored by each player during each turn. Highest total after 4 rounds wins (3 rounds for 3 players).

In partnership play, the higher combined score of the two players wins.

Variations:

You can spice up this strategy version of RunnuRound by adding variations given for the Random Scramble version:

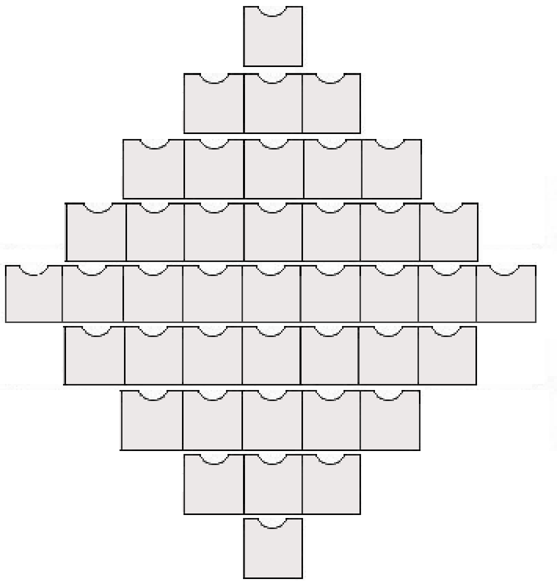
- “Wild” tiles
- “Elevator” wrap
- Vertical wrap

Alternate boards

For those with a taste for the extreme, play RunnuRound as either of the optional board layouts below.

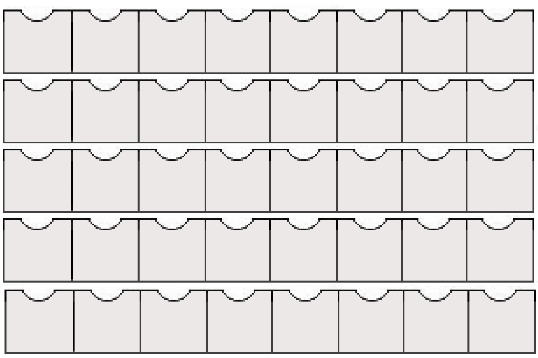
On the diamond-shaped layout, look for runs of 3 in all four directions—horizontal, vertical and along both diagonals.

Add one wild tile and mix the colors in any random order before arranging the diamond and revealing the numbers row by row. How fast can you make your predictions?



The 5x8 rectangle below is another mixed-color layout for the 40 number tiles. Here again the colors don't matter, only the numbers.

After fitting the tiles face-down into the rectangle, flip over one row at a time. How many runs would you predict before all the numbers have been revealed? What do you think is the probability of finding more runs when there might be several duplicate numbers in any row?



Solitaire Quests

The No-Runs problem
Arrange the 40 number tiles in a 4x10 rectangle to that there are no runs at all, neither horizontally nor vertically.

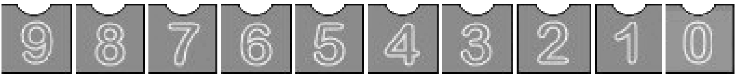
The Most-Runs question
With all 40 number tiles face down, mix them well and then arrange the 4x10 rectangle in rows by color. Remove the last two tiles in each row and set them aside. Turn all the tiles face-up. As quickly as possible, insert the set-aside tiles into their respective rows at any point you choose so as to maximize your runs. What is your highest score?

The Latin array
Arrange the 40 number tiles in a 4x10 rectangle so that there are no two tiles of the same number in any row, horizontally nor vertically. Furthermore, no two tiles of the same color should occur in any diagonal row.

Magic sums
Here's a challenge for the kids in the family. Place all 40 tiles into the 5x8 rectangle so that all 8 vertical columns add up to a different sum, and the eight sums are consecutive integers. Zeros count as zero. The total value of all the tiles is 180. For extra challenge, have no two of the same number in any row.

Acknowledgments

We're very grateful to these individuals whose advice, suggestions and contributions were an invaluable addition to the development of RunnuRound: Paul J. Campbell, Ph.D., Beloit College; Ashish Vikram, Bangalore; and Eric Bare, Miami.



RunnuRound™

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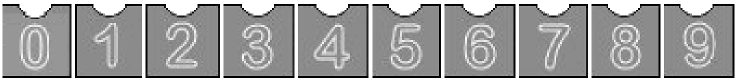

Kadon Enterprises, Inc.
1227 Lorene Drive, Suite 16
Pasadena, MD 21122
www.gamepuzzles.com
kadon@gamepuzzles.com

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