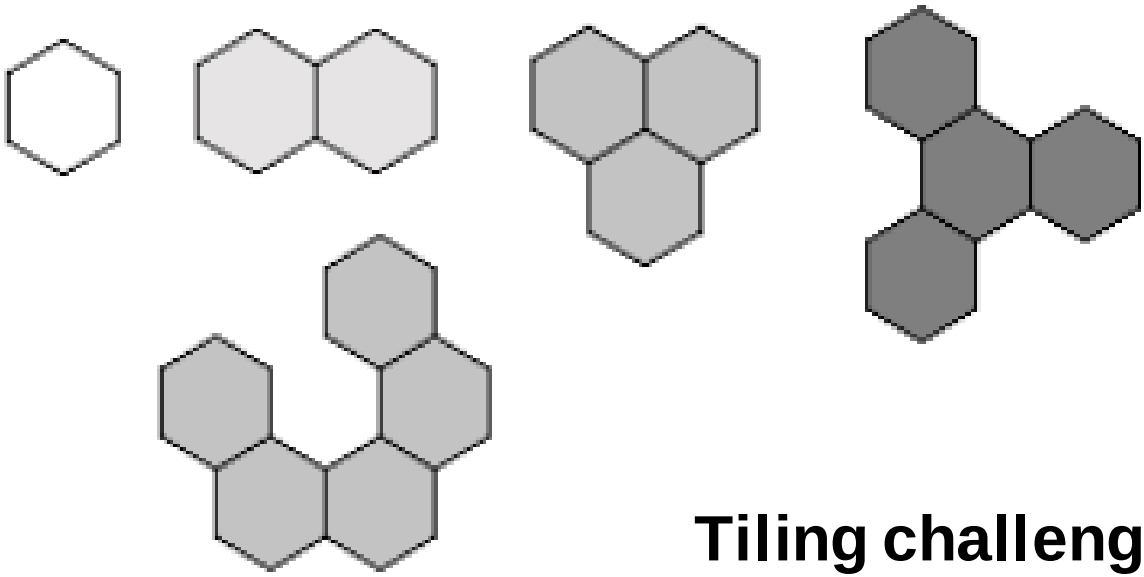
1 to 4 players
Ages 10 to adult

Hexnut™

Myriads of explorations with 5 sizes of polyhexes



Tiling challenges
Puzzle designs
Game rules

A product of
Kadon Enterprises, Inc.

Hexnut™ *is a trademark of Kadon Enterprises, Inc., for its set of polyhexes orders 1-5.*

Table of Contents

The beautiful Hexnut acrylic tiles are lasercut by
Kadon Enterprises, Inc.— Pasadena, MD 21122
www.gamepuzzles.com

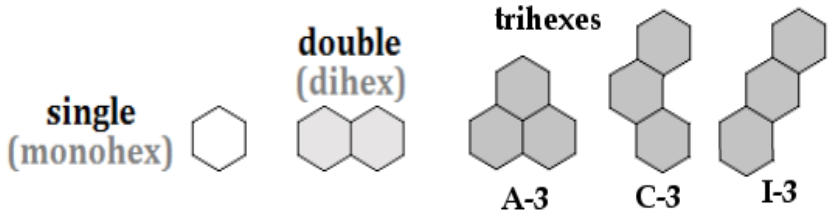
INTRODUCTION

The **HEXNUT** set is made up of the complete group of shapes made by joining equal-sized hexagons evenly on their edges, using from 1 to 5 hexagons. They are known in recreational mathematics as *polyhexes*. Depending on how many hexagons a shape contains, it gets a more specific name.

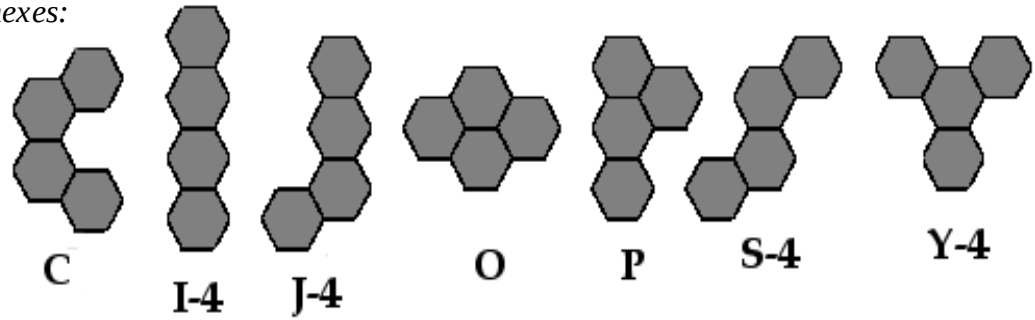
The polyhexes in the Hexnut set consist of: 1 monohex, 1 dihex, 3 trihexes, 7 tetrahexes, and 22 pentahexes. Kadon also produces a size-compatible companion set of the 82 hexahexes under the name of Hexnut II. One of the hexahexes contains an enclosed hole. Enumerations of higher orders can be found on the Web. Just search for *polyhexes*.

The 12 smallest pieces, plus 5 selected pentahexes, form a separate introductory set that Kadon makes under the name of Hexnut Jr. It is suitable even for younger players and has a small book devoted to just those pieces. That book is included with Hexnut.

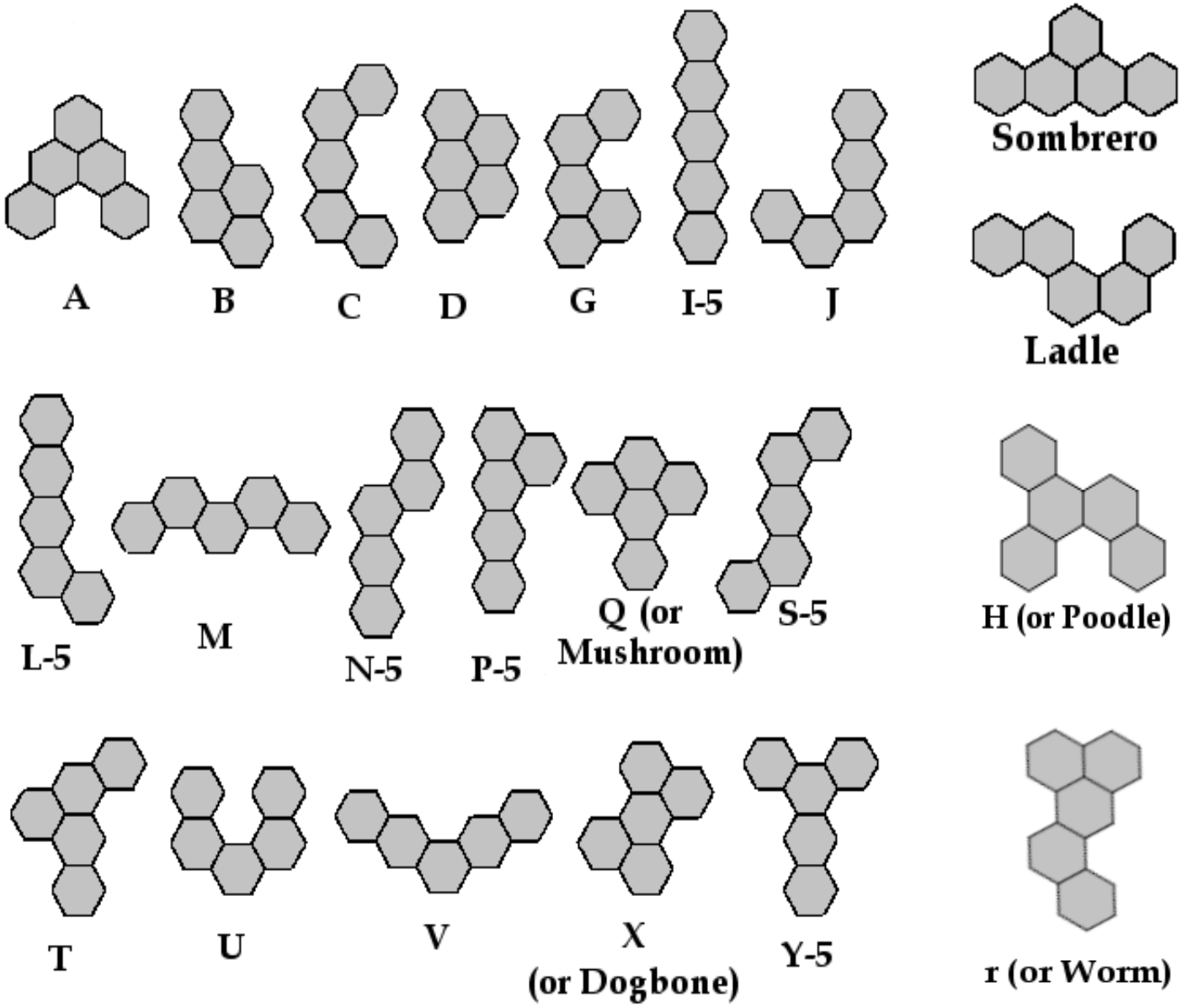
Here are all the Hexnut tiles with their accompanying names. Pieces of the same size have their own color.



The tetrahexes:



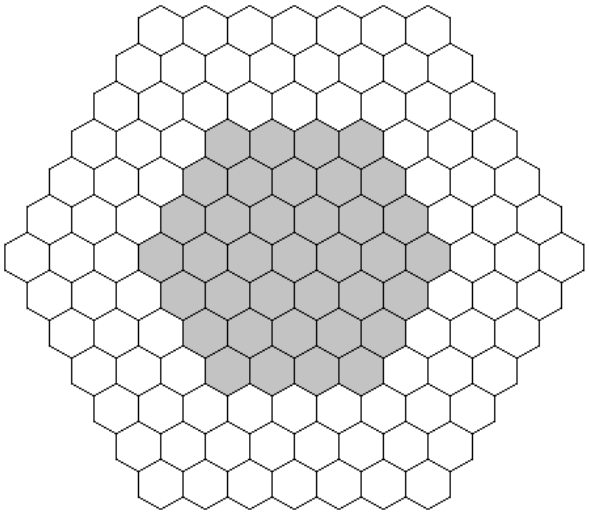
The pentahexes:



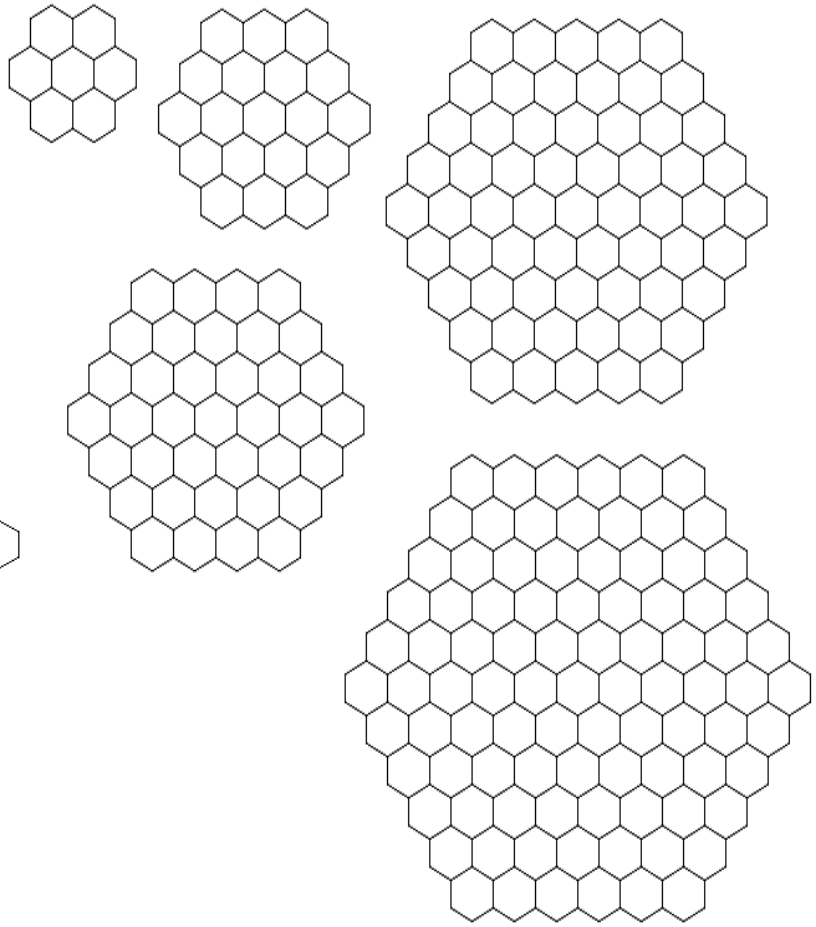
A TREASURY OF PUZZLE FIGURES

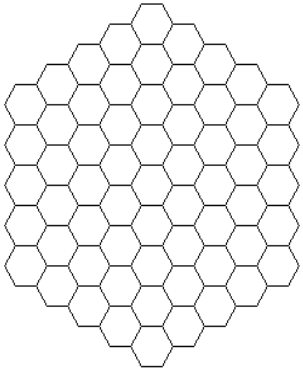
Polyhexes are among the most versatile shapes with which to construct patterns. Nature has the same idea with beehives and honeycombs. We present here a few hundred challenges accumulated over more than 25 years of research and puzzling. Start out easy, then more complicated, and finally as advanced as imagination allows. They all have solutions, though not included in this book. *Happy puzzling!*

Hexagons



Fill the shaded area with the smaller pieces.

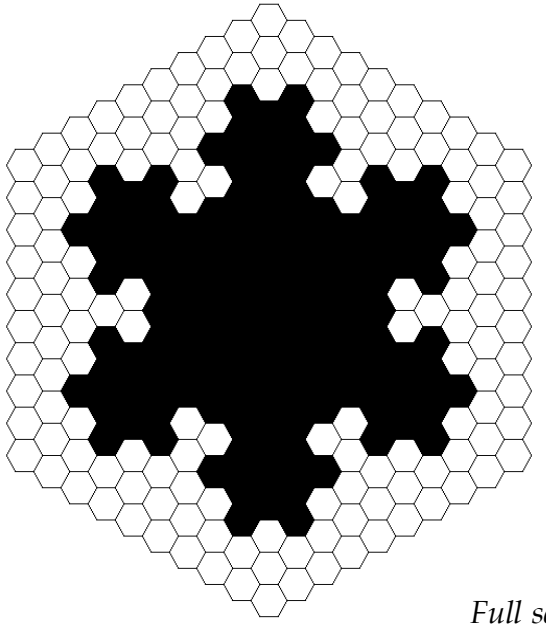




with 11 symmetric pentahexes + two trihexes

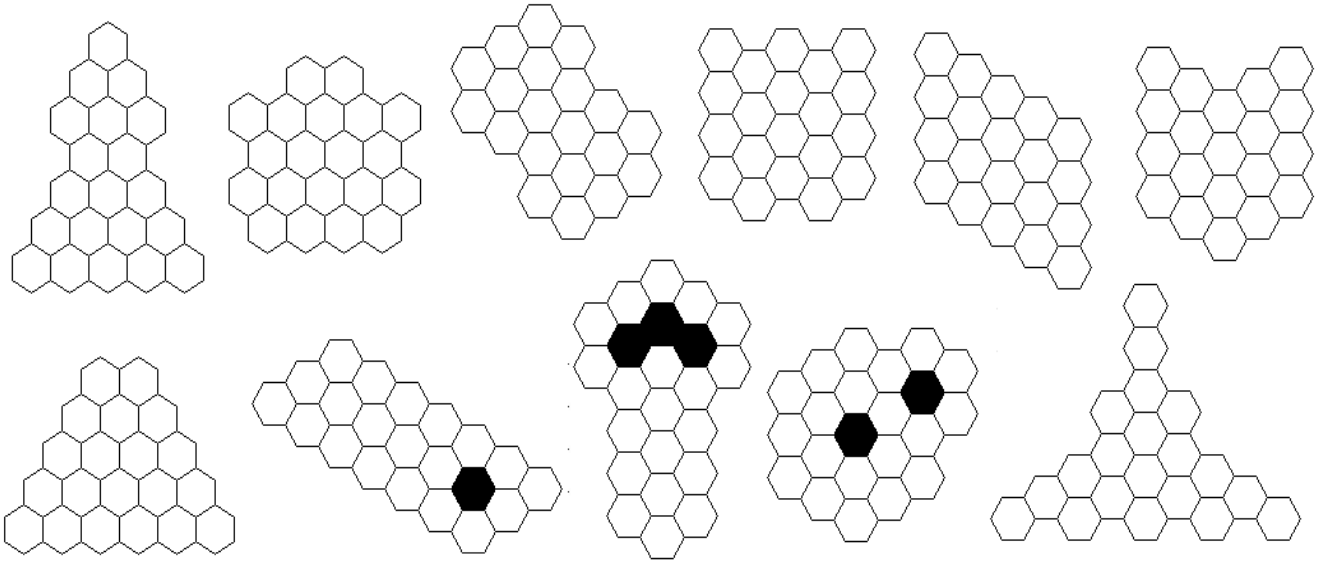
---

with 11 asymmetric pentahexes + two trihexes



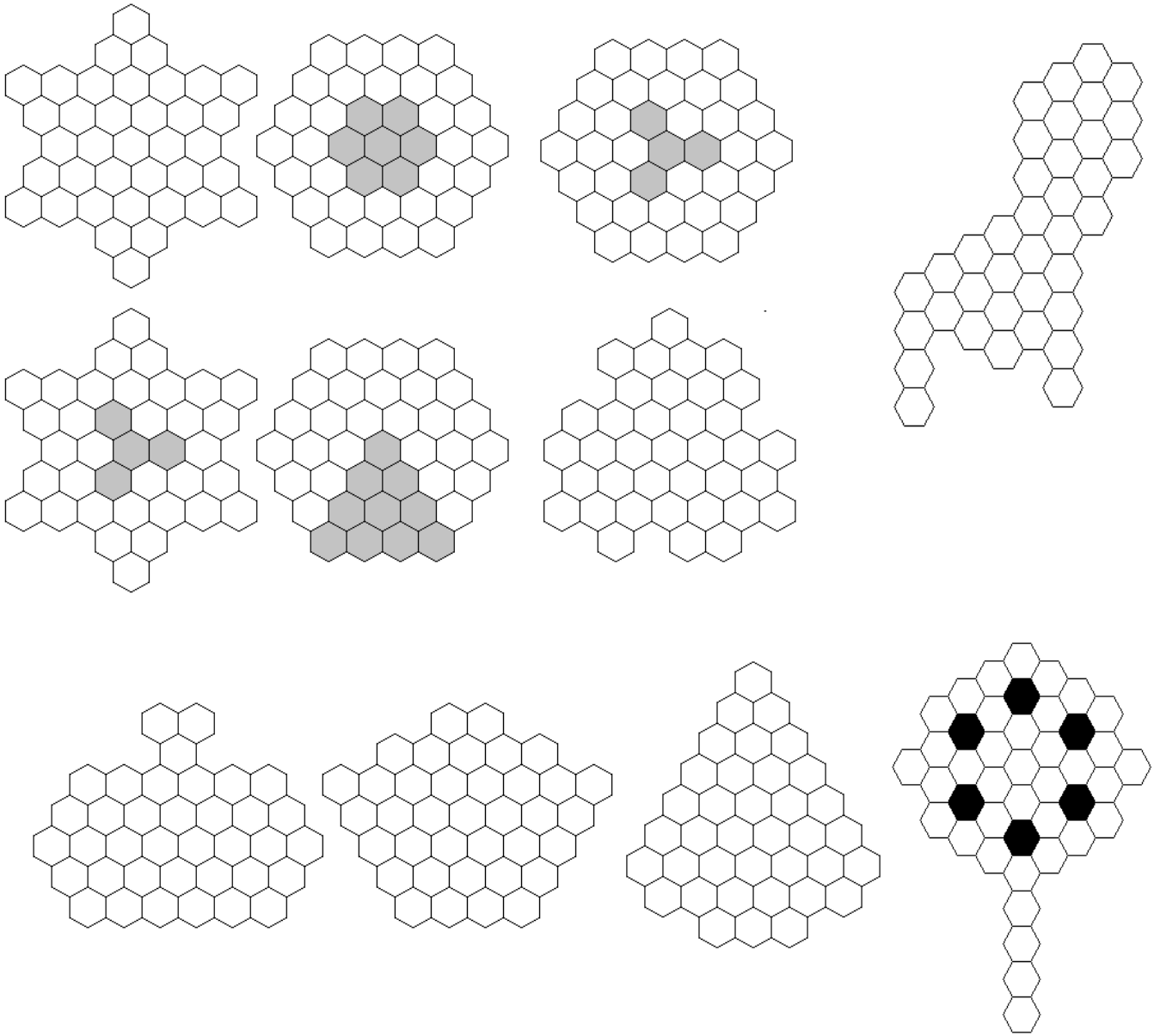
Full set, snowflake hole

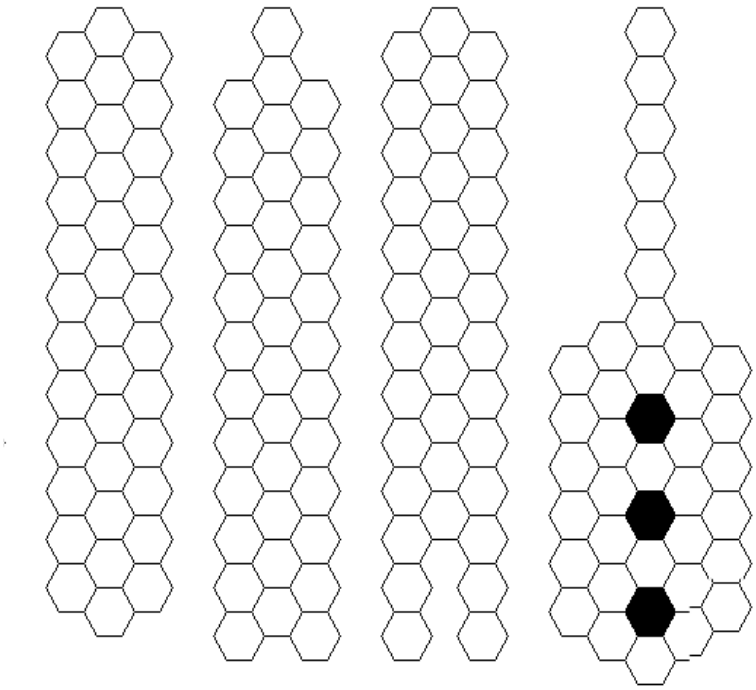
Tetrahex symmetries

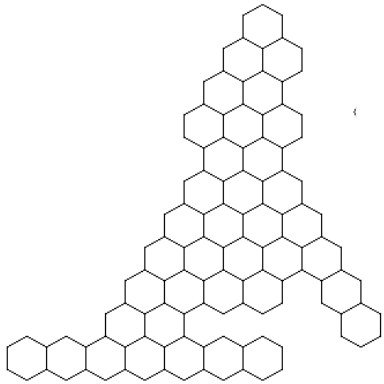


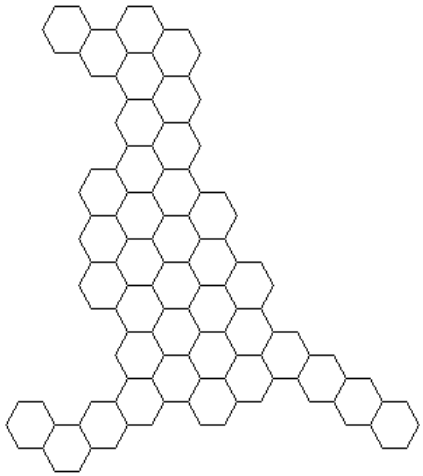
Solve each of these symmetrical shapes with just 5 of the tetrahexes.

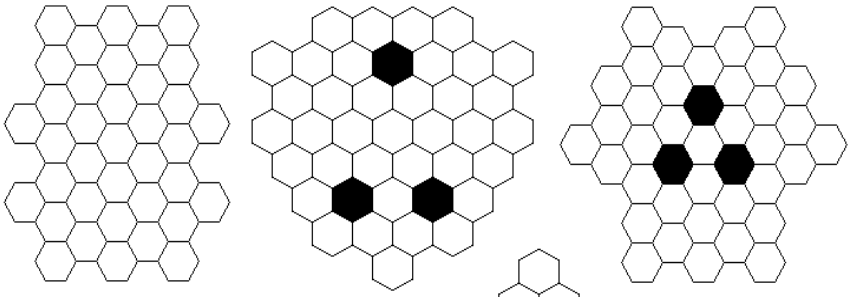
Warm-ups with all Trihex and Tetrahex tiles

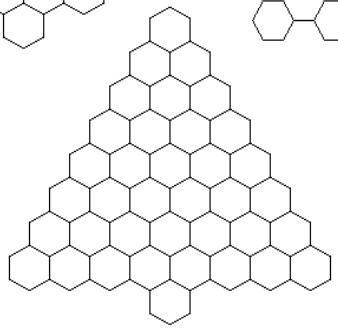


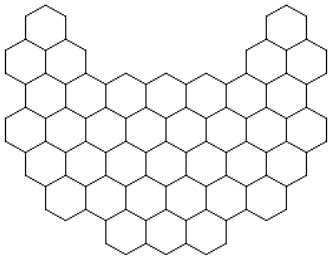


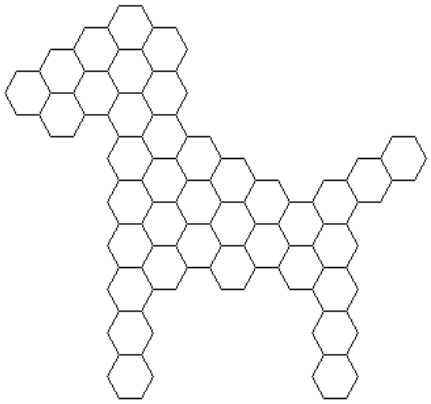




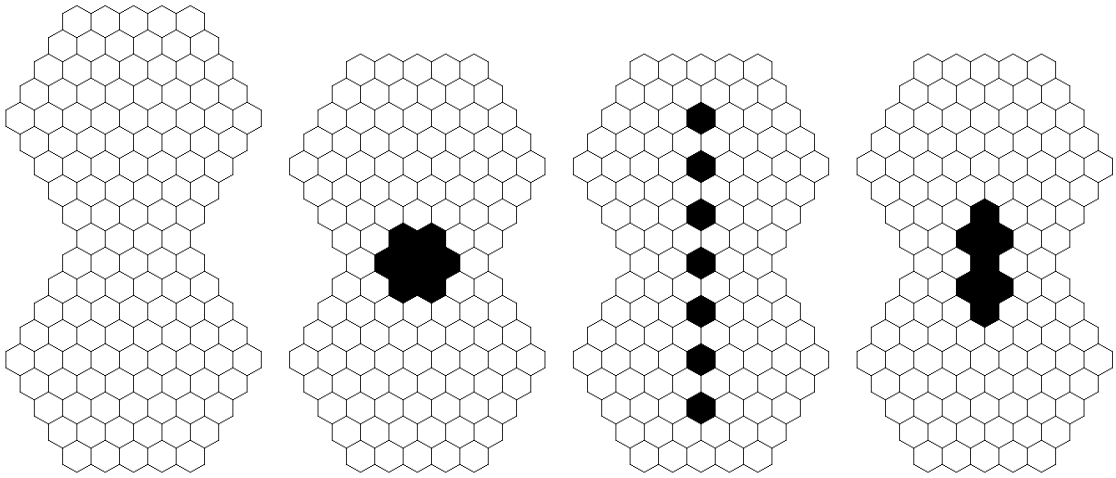






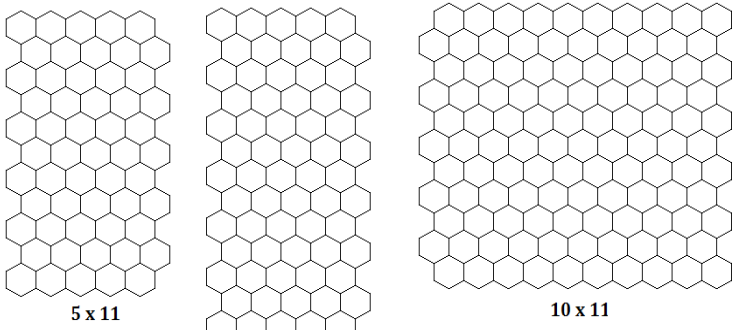


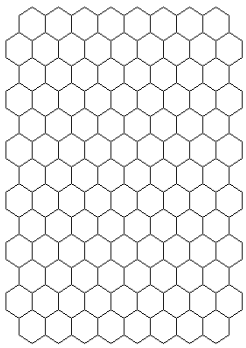
Hourglass parade

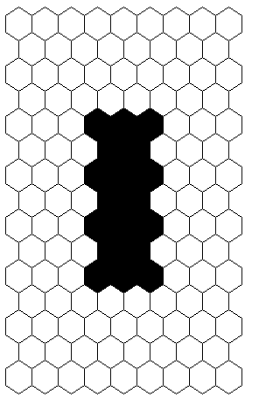


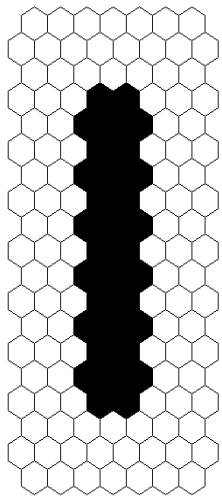
Tapestries…

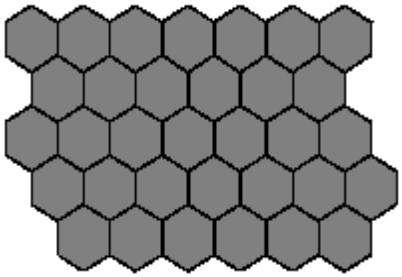
Use as many pieces as needed to build these, the fewer the better (more pentahexes)..





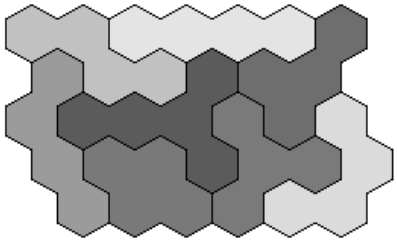




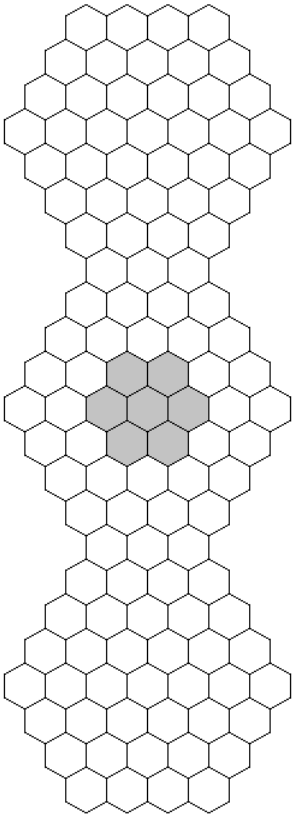


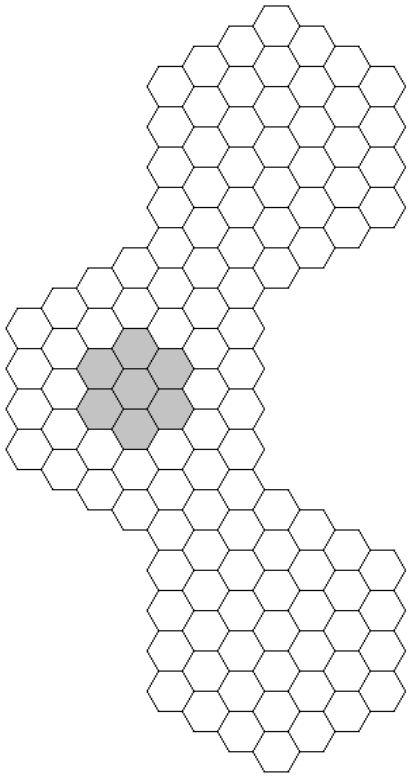
Captive pentahexes --

22 challenges: build this shape 22 times with the 7 tetrahexes, each time with a different pentahex at its center. Here's a sample:

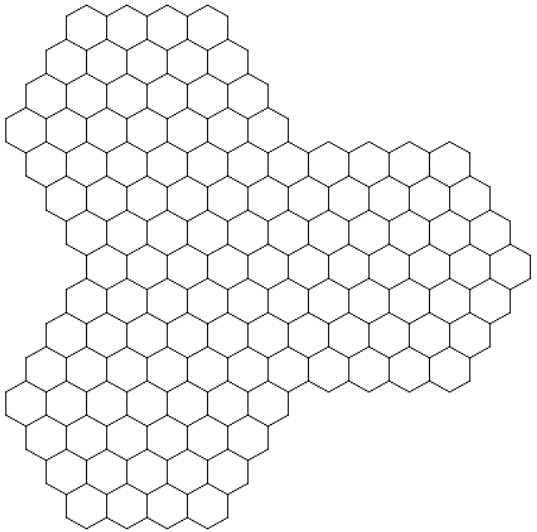


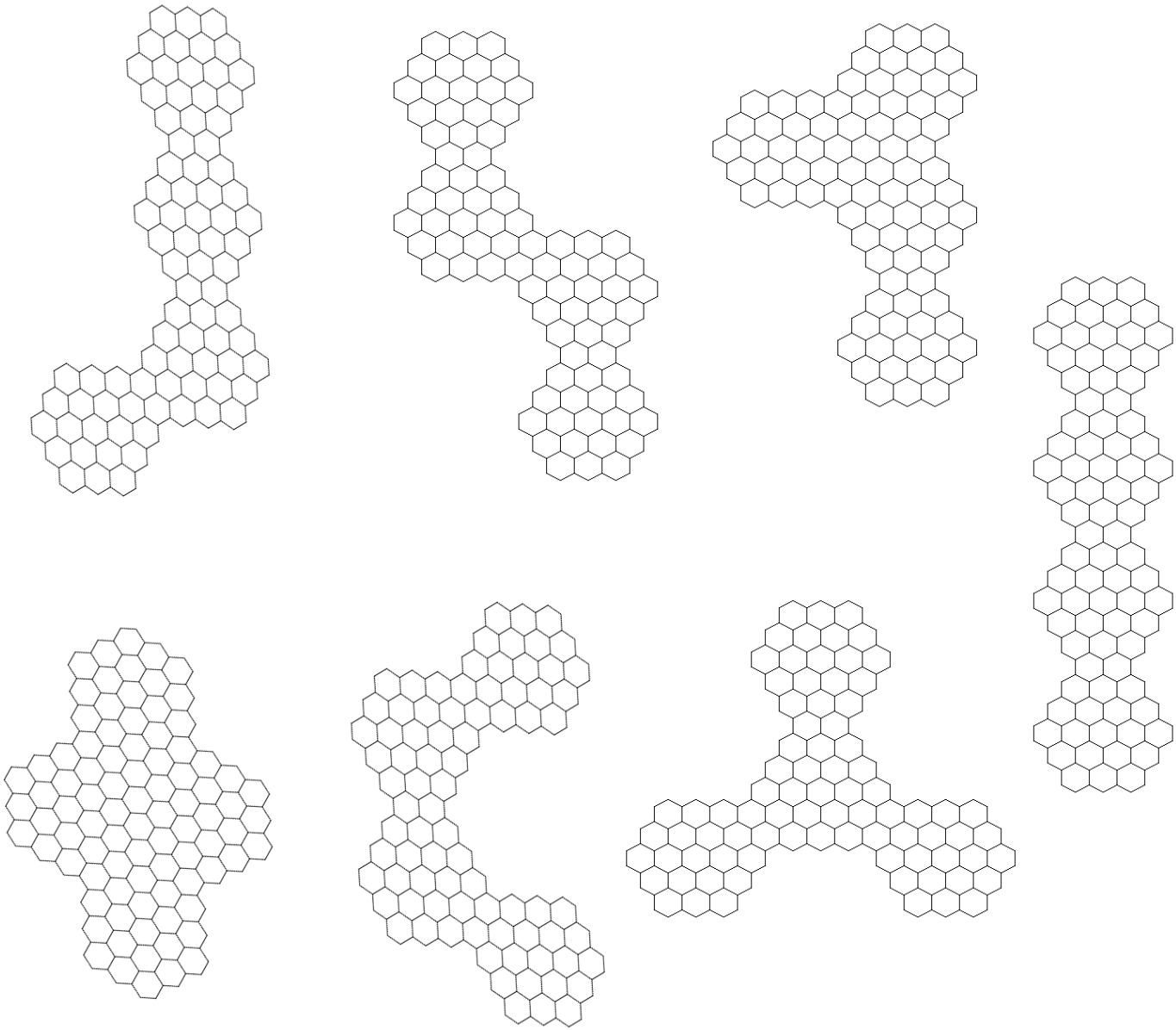
Modeling polyhexes



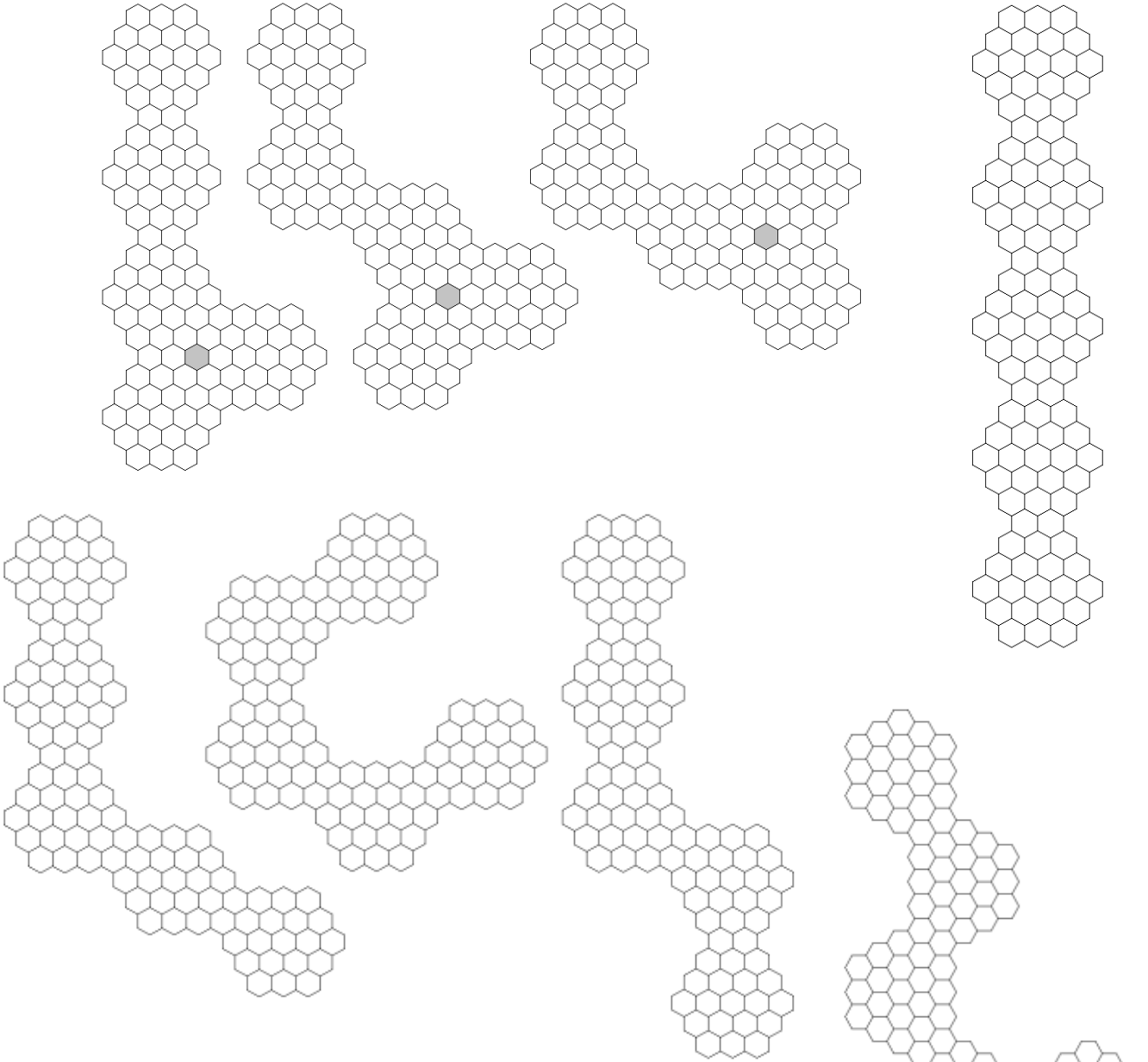


The three trihexes, enlarged four times.

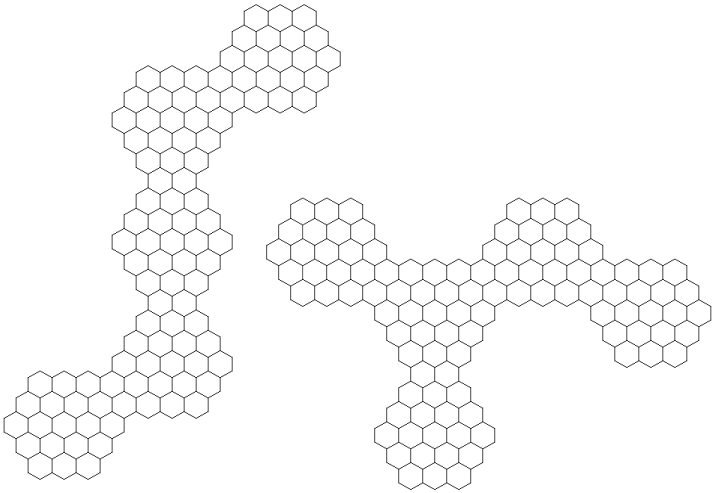


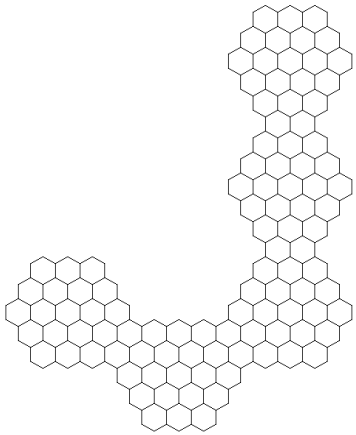


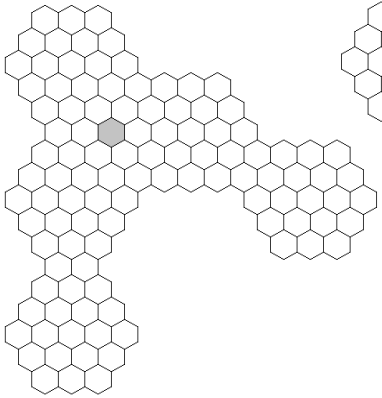
The seven tetrahexes, enlarged 3 times.

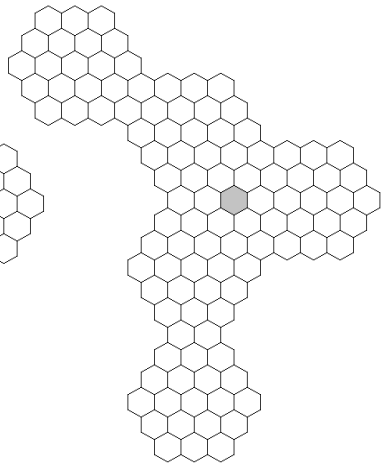


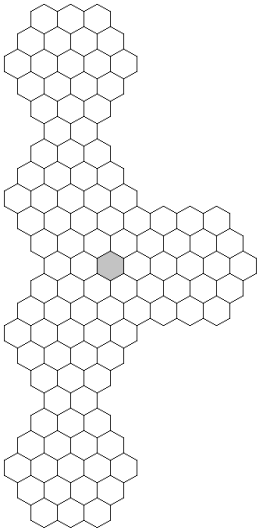
The 22 pentahexes enlarged 3-fold

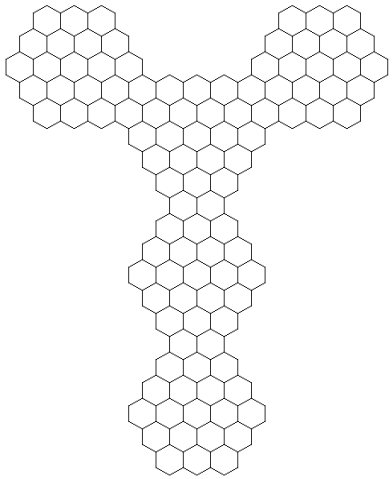


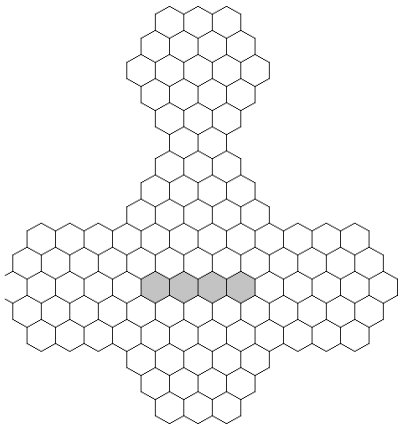


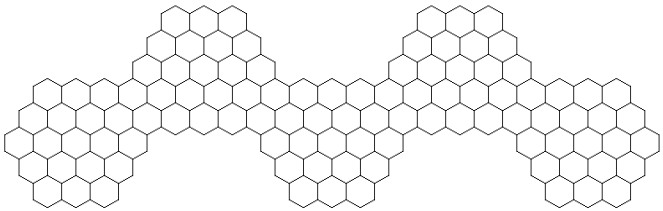


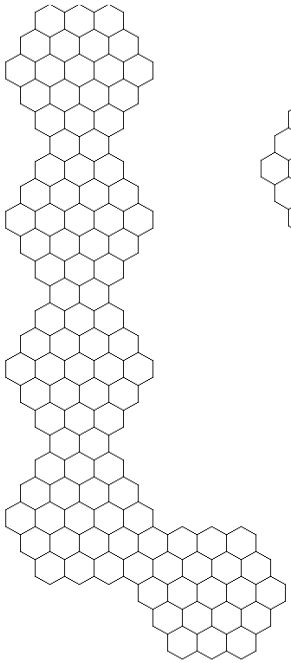


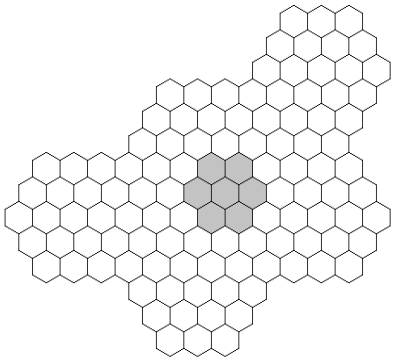


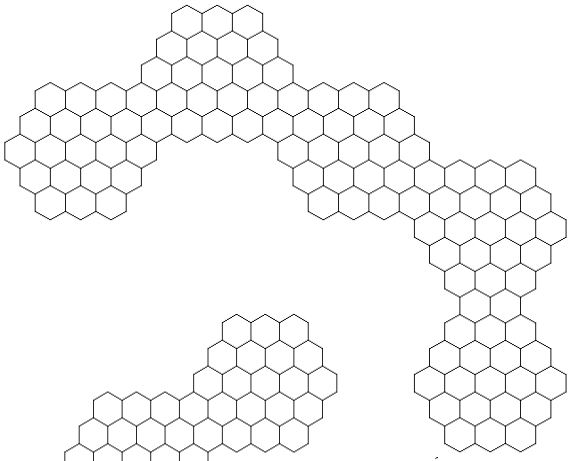


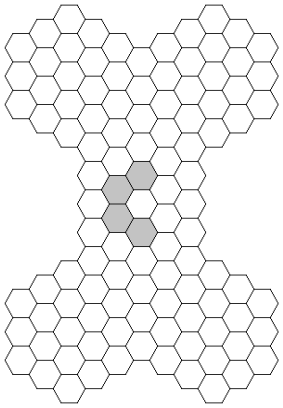


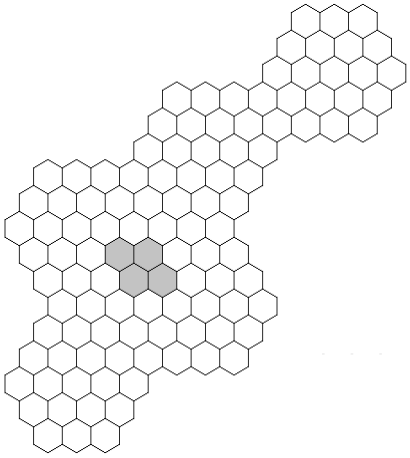




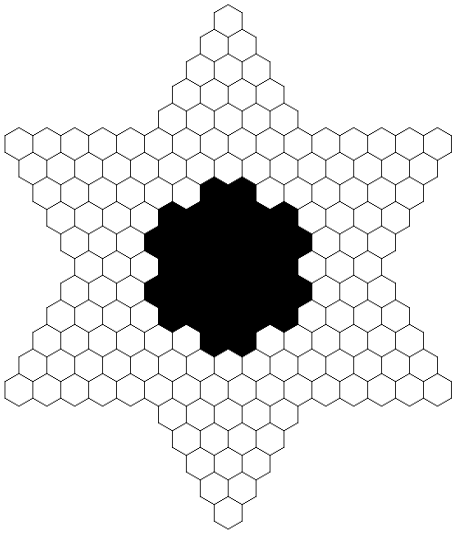


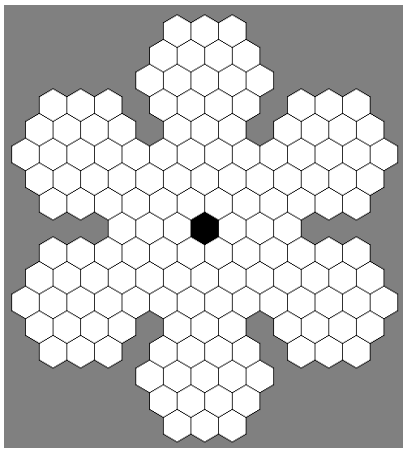


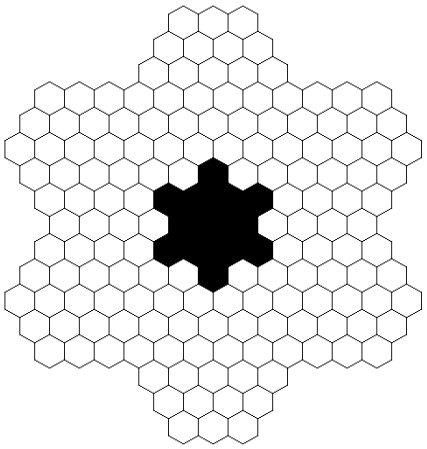


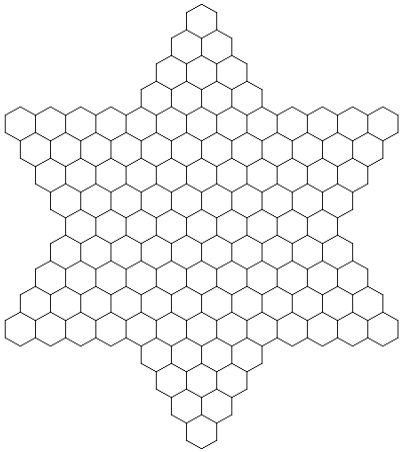


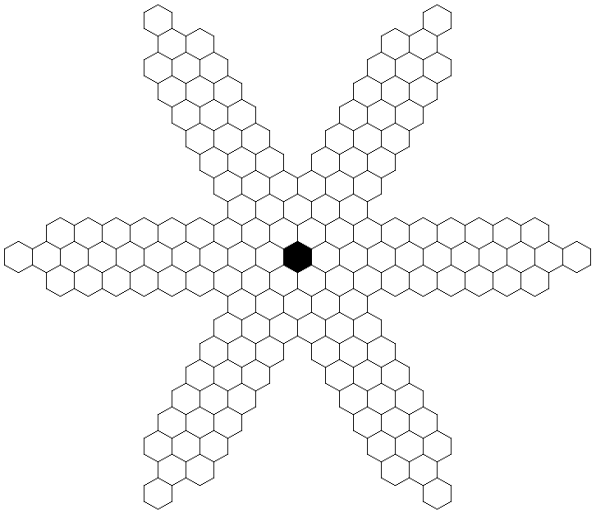
Snowflakes

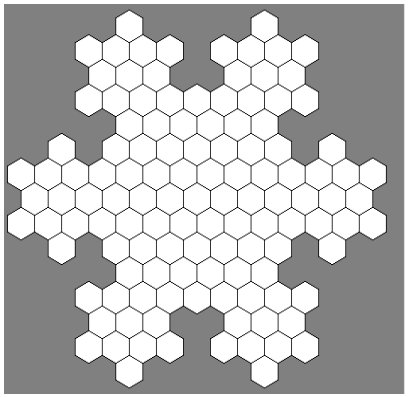




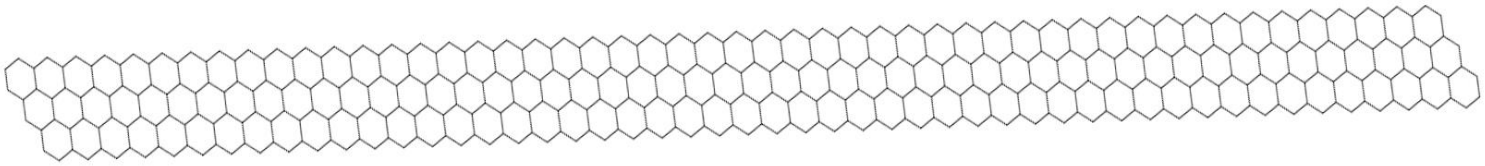


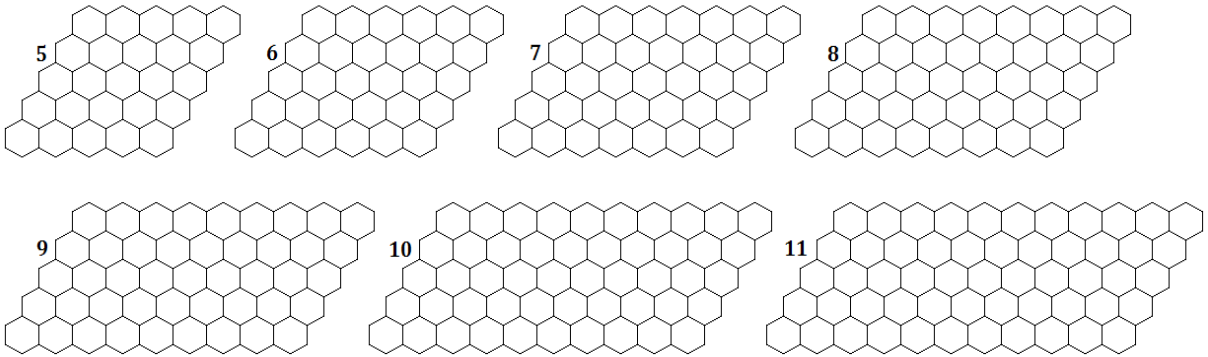


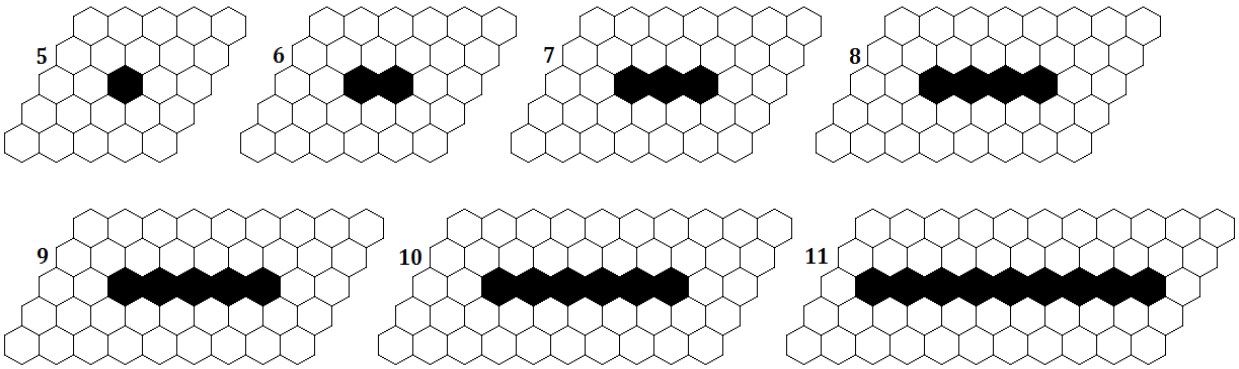


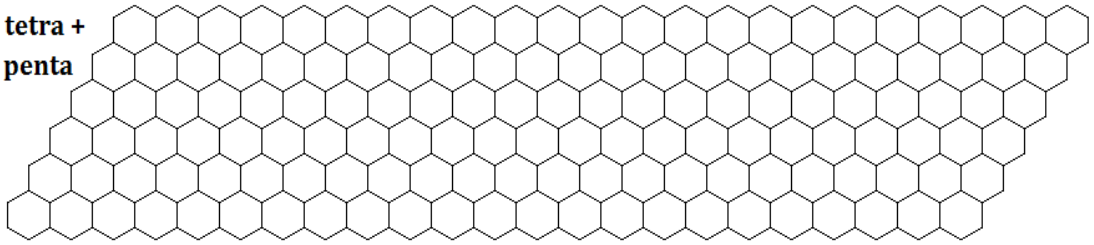


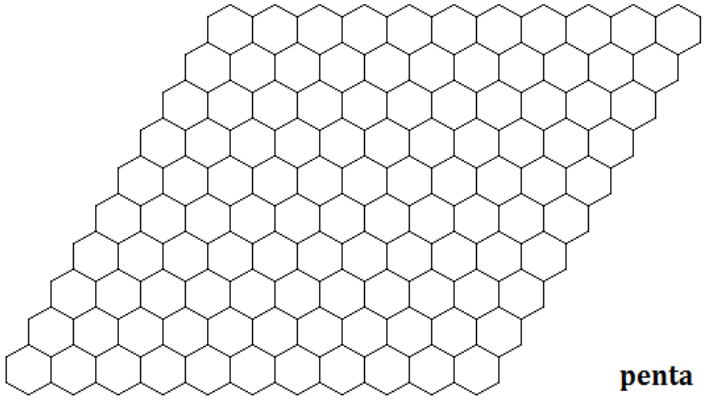
Parallelograms — *try other sizes, too.*



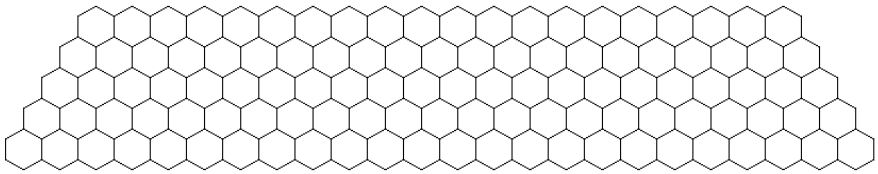


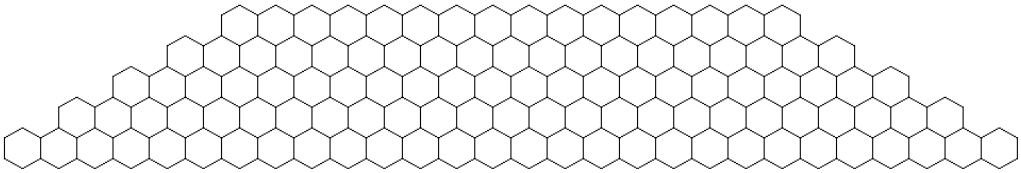

5
6
7
8
9
10
11

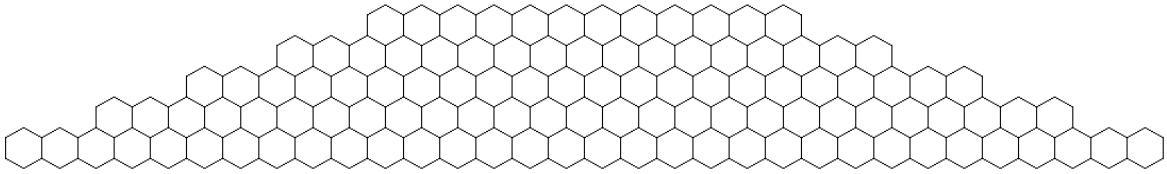

tetra +
penta

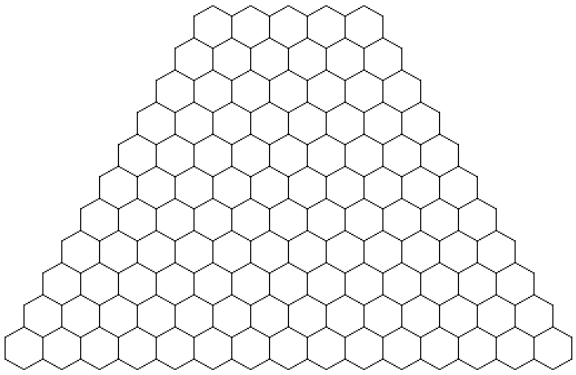

penta

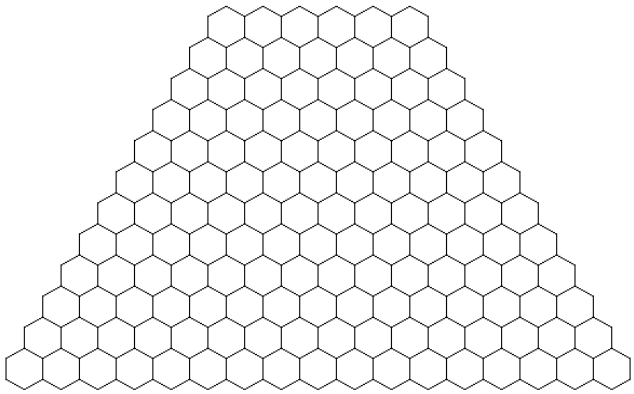
Trapezoids — *experiment with other sizes, too, from smallest to largest.*



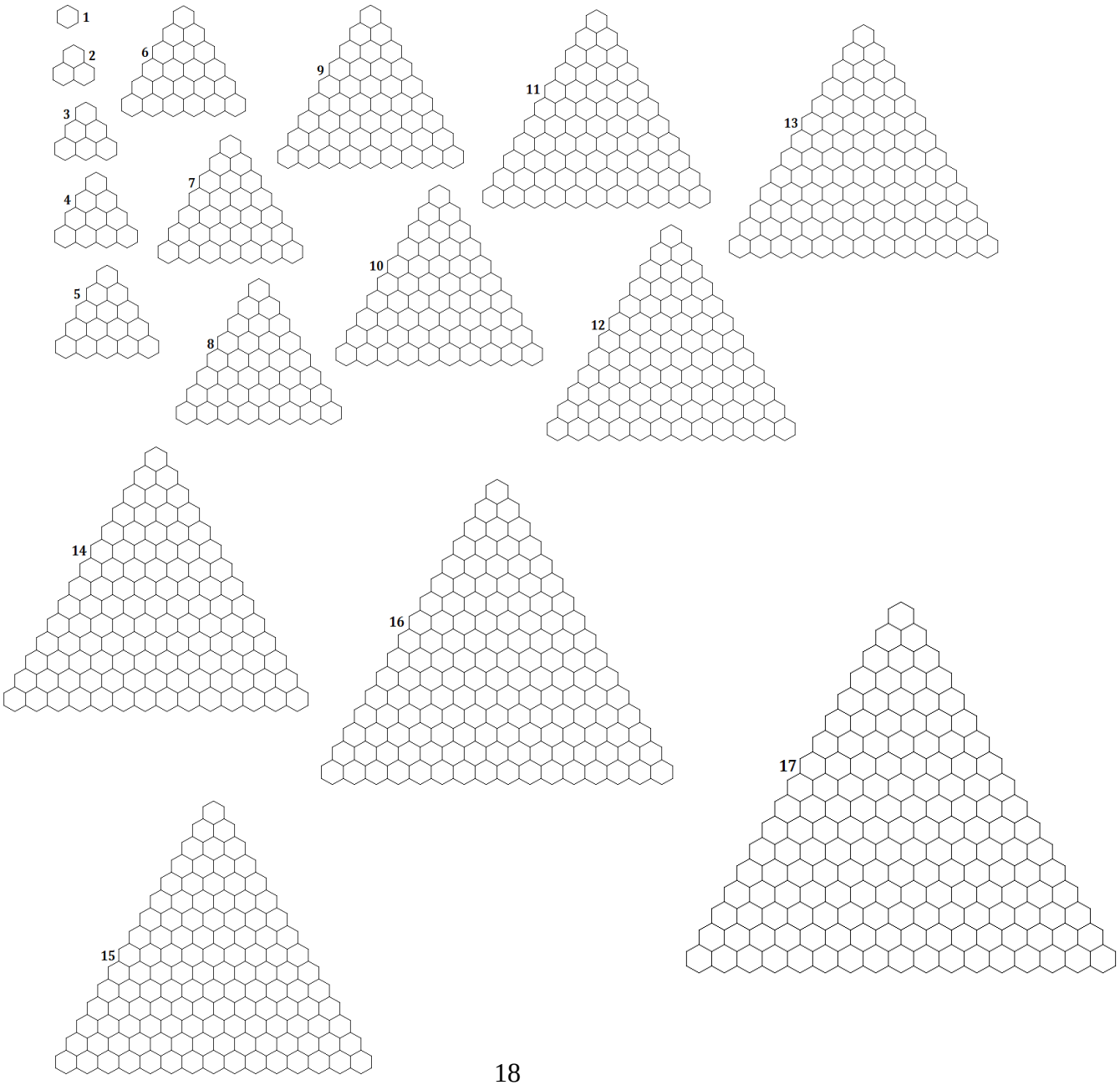


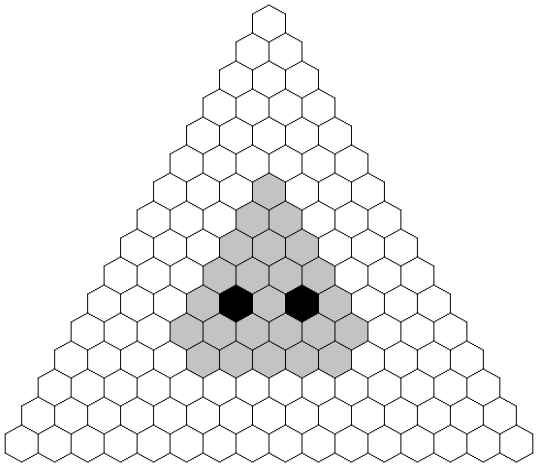


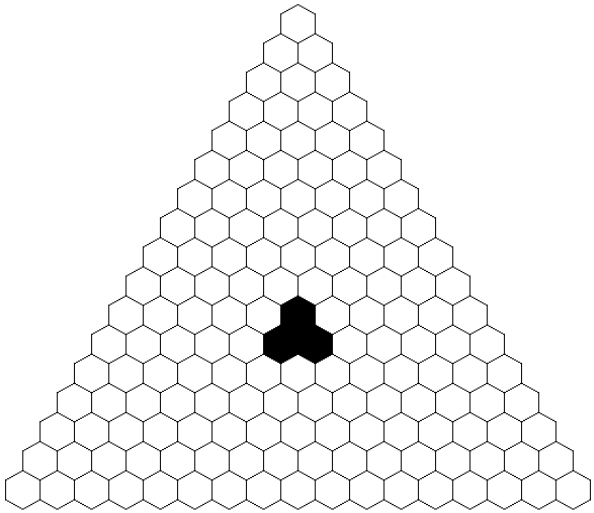




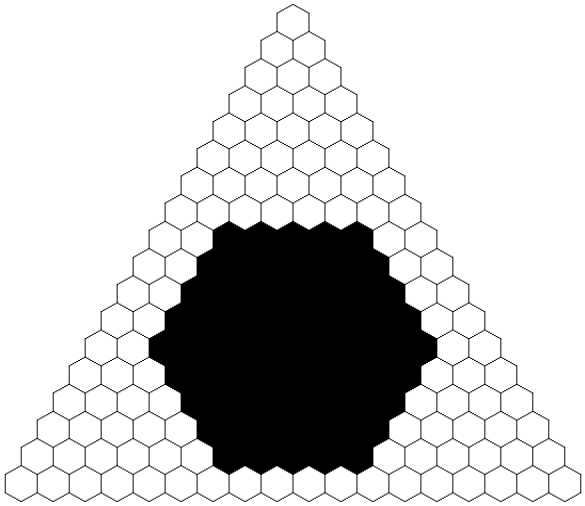
Equilateral triangles

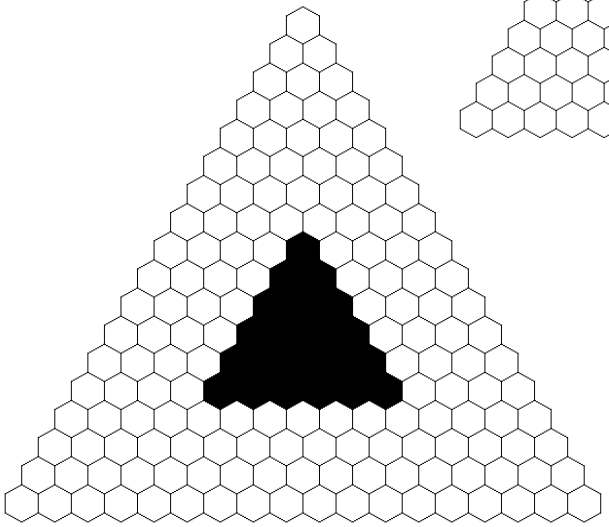


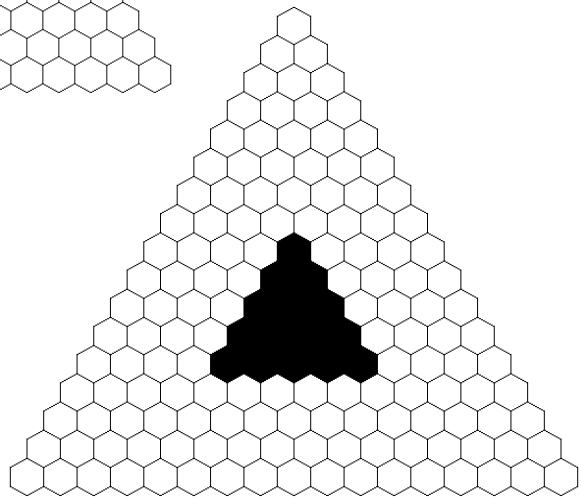




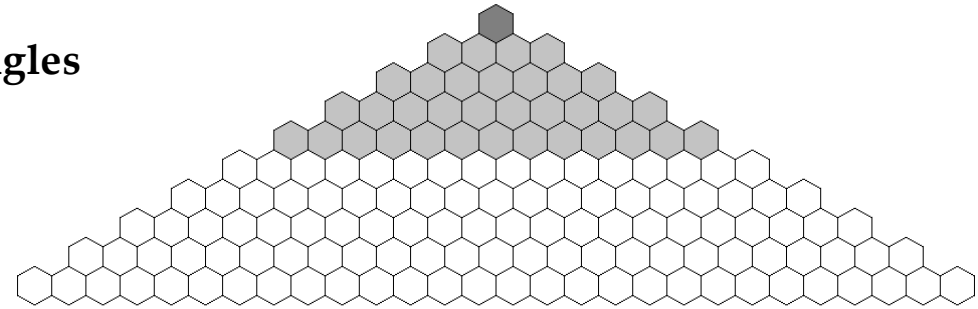
Use tetrahexes in shaded area.
Black spaces are holes.

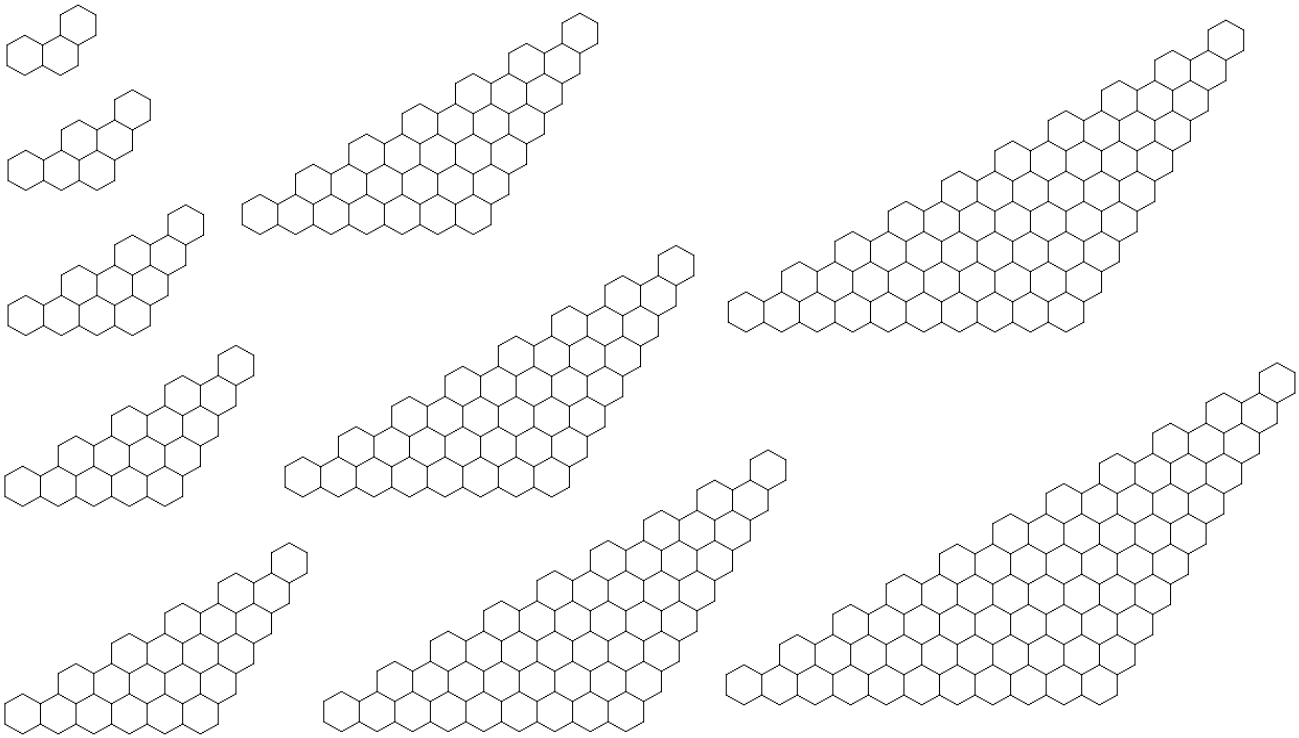






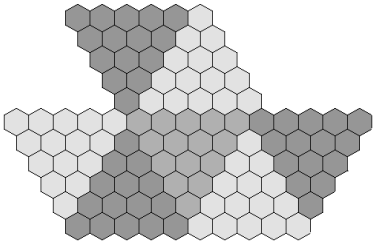
Isosceles triangles

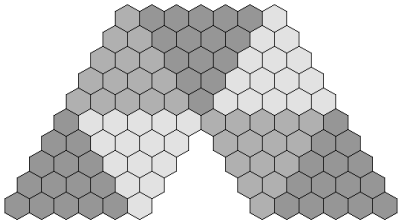


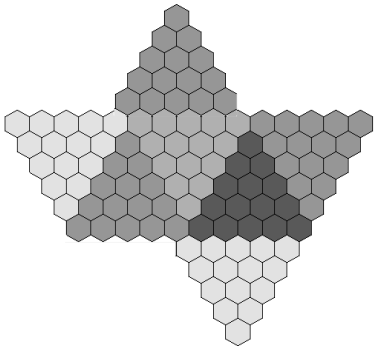


Explore other sizes and shapes.

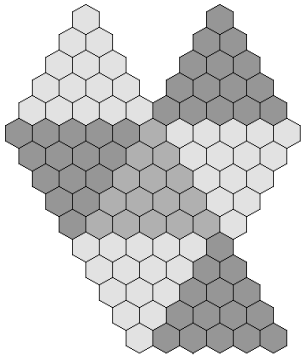
Fancy figures

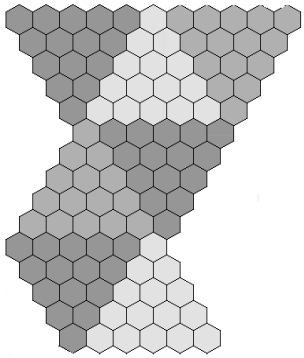


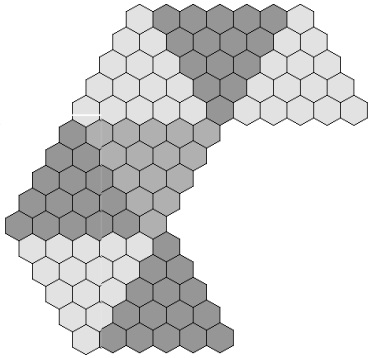


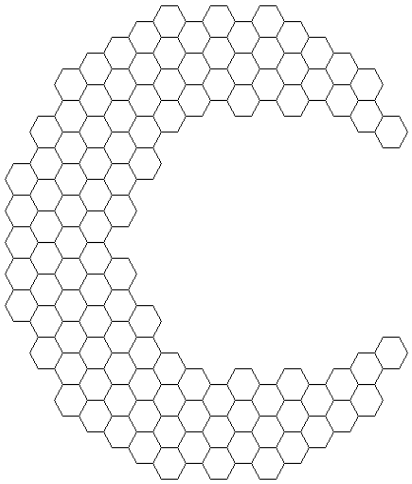


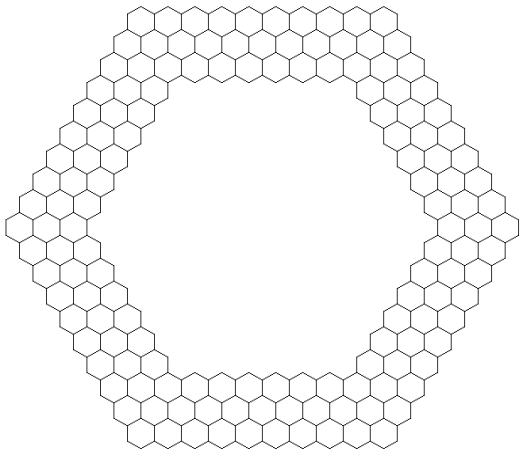
These six figures are the shapes of 7 equilateral triangles joined (called heptiamonds). Solve them with 21 pentahexes. There are 24 all-different heptiamonds. Can you form the other 18 and solve them?Shading is to show structure, not to divide pieces.

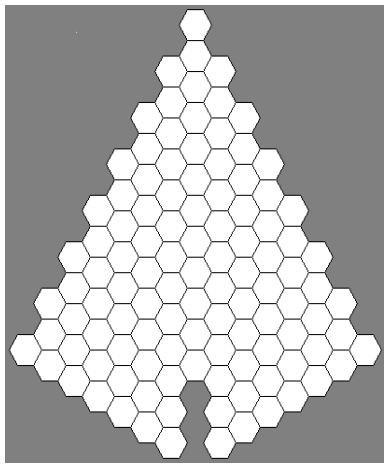




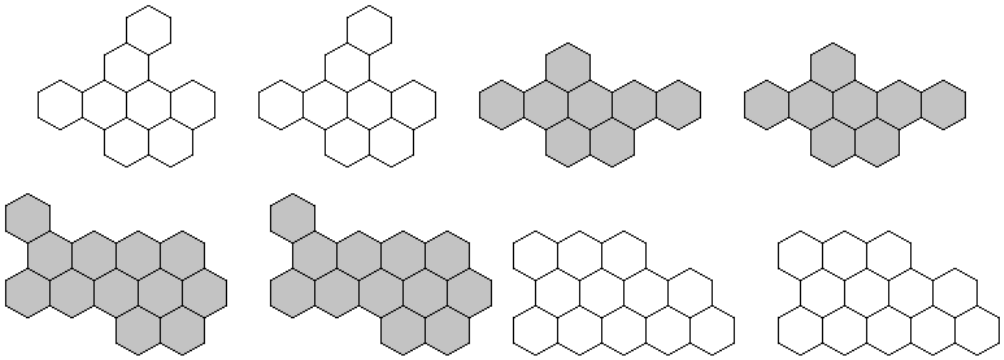




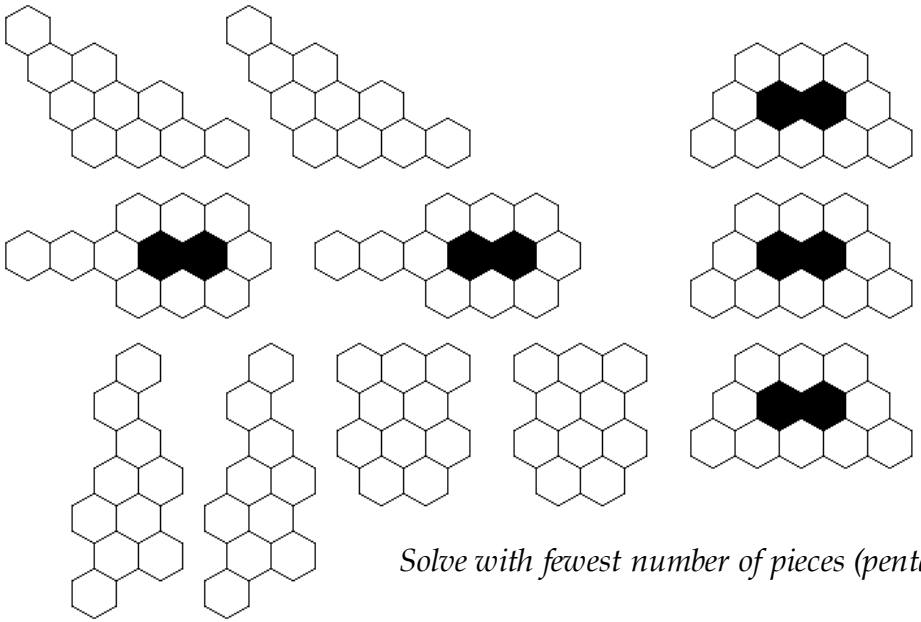




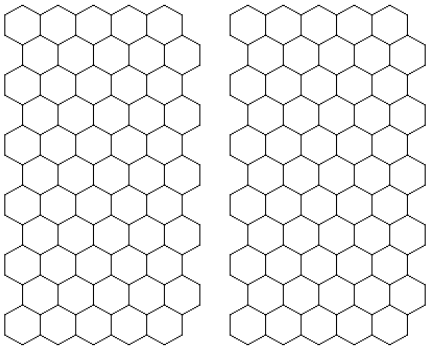
Twins and more

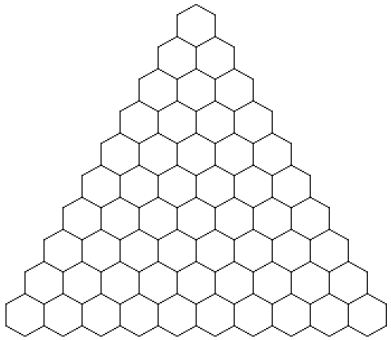


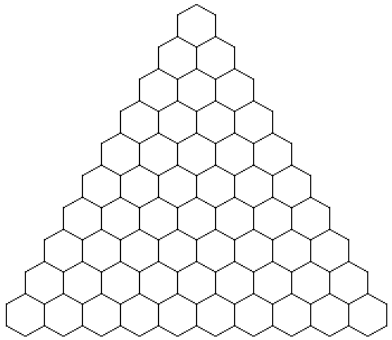
There are dozens of ways to join pairs of tetrahexes to form two congruent shapes simultaneously, as exemplified by the top row. When you join trios of tetrahexes, the number of shapes skyrockets into the *hundreds*, just two of which are shown in the bottom row. How many can you find?

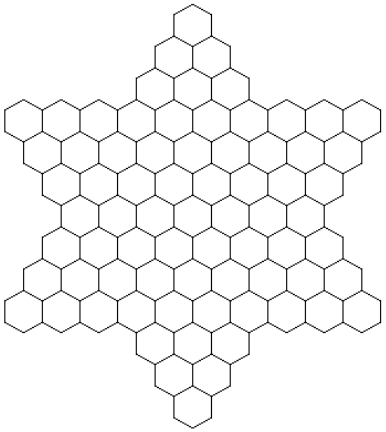


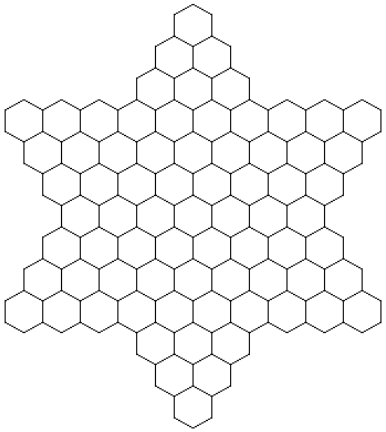
Solve with fewest number of pieces (pentahexes)

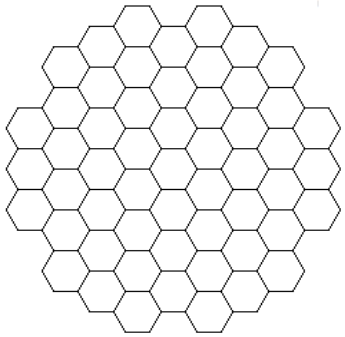


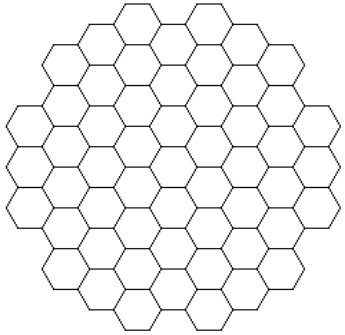


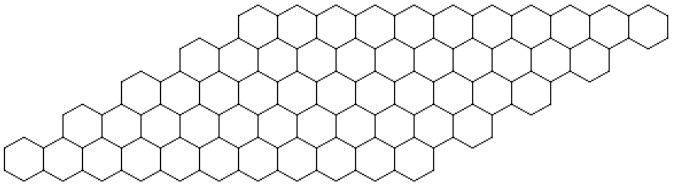


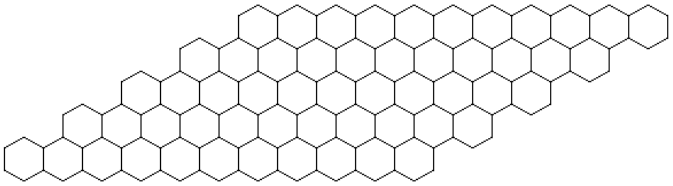


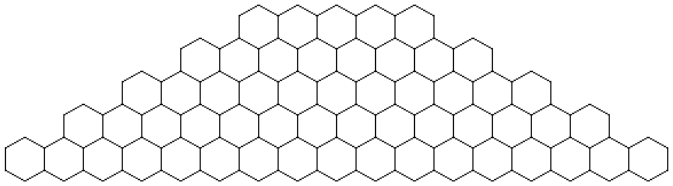


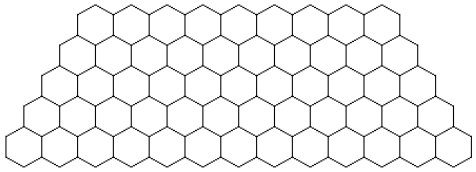


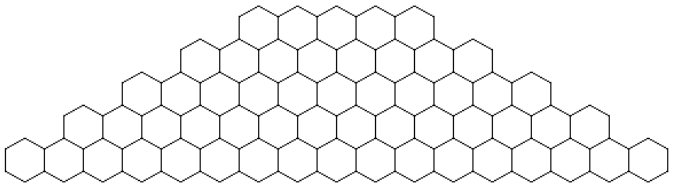


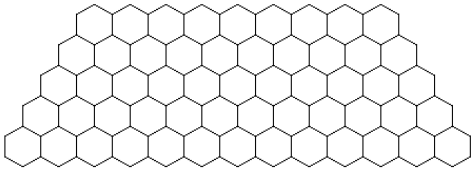


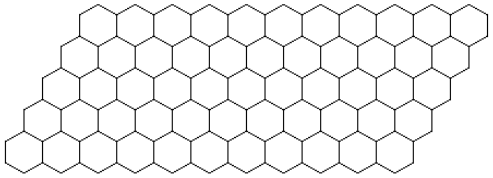


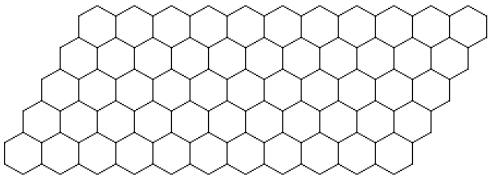


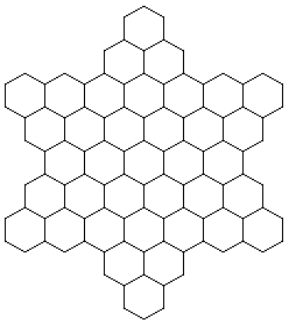


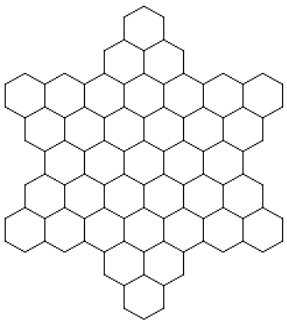


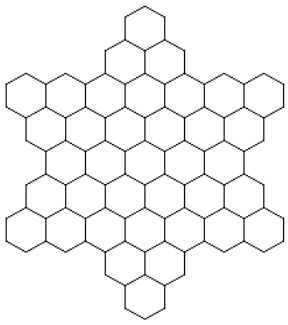


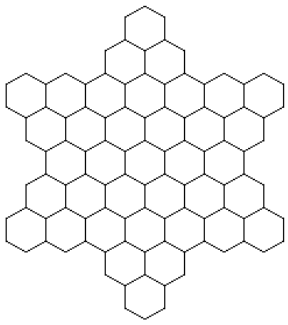




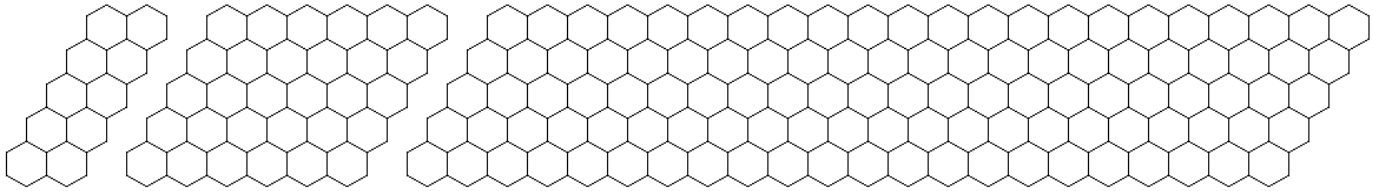




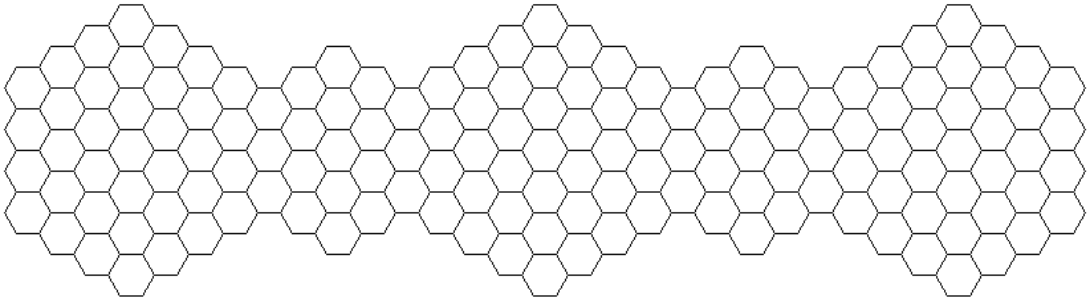


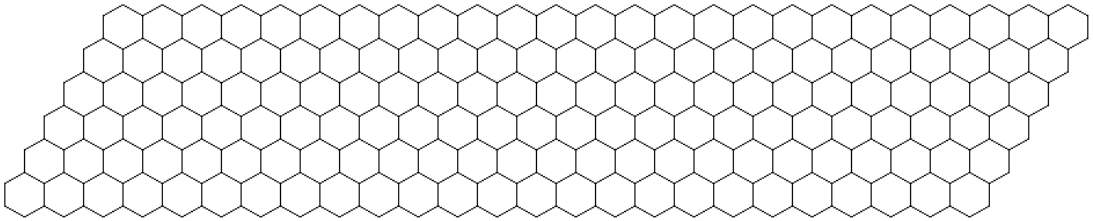


The full set

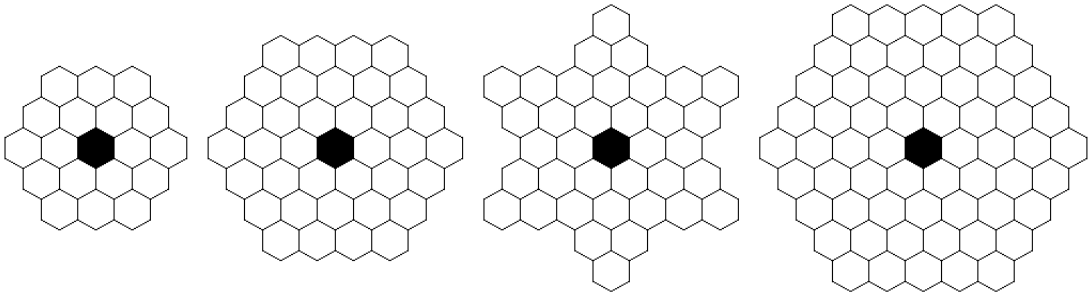


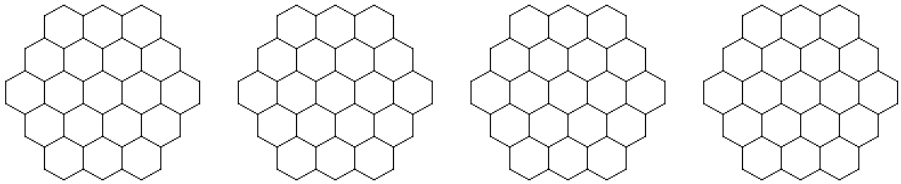
5 x 2 (tri, mono) 5 x 6 (tetra, di) 5 x 22 (penta)

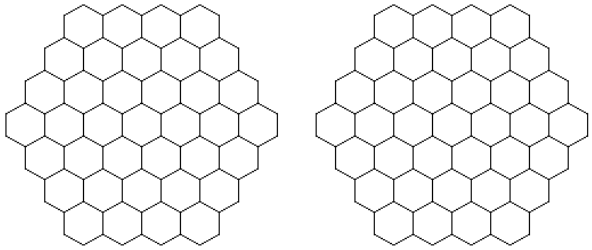




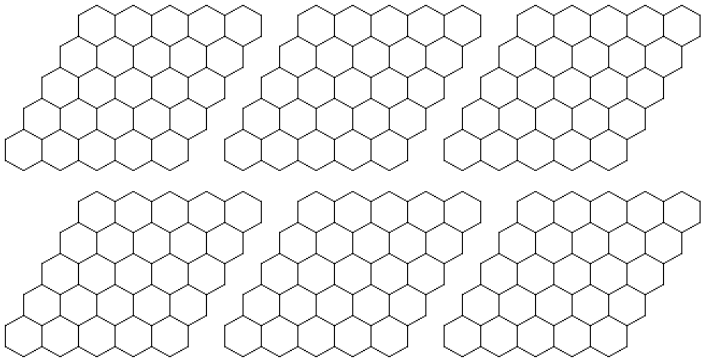
Special Challenge: pack the 6 x 25 parallelogram grid with the entire set of polyhexes, orders 1 through 5, such that the mono-, di-, tri- and tetrahexes neither touch one another nor the border of the figure. It is possible!



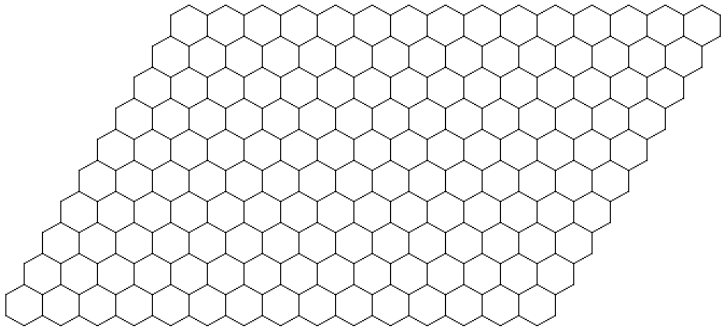




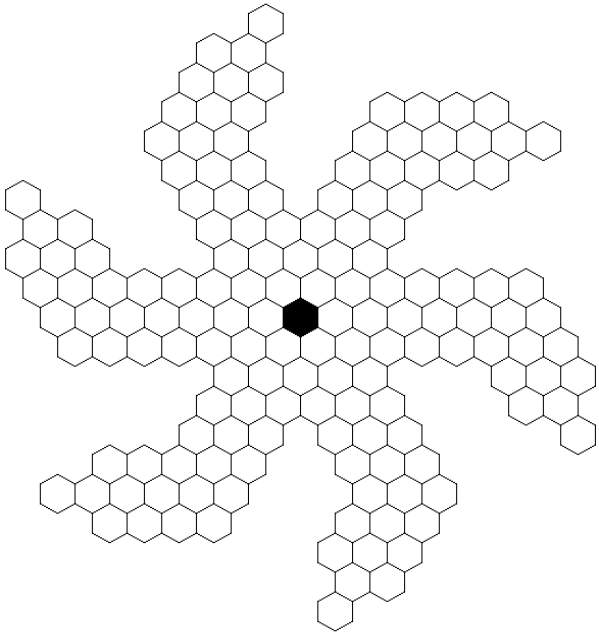
Build 4 small and two large hexagons simultaneously.

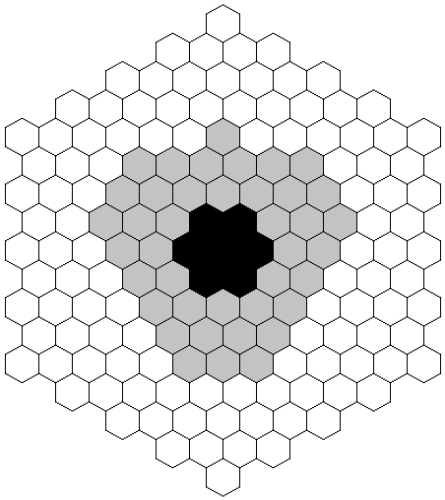


Build six 5x5 rhombuses simultaneously, using up the whole set.

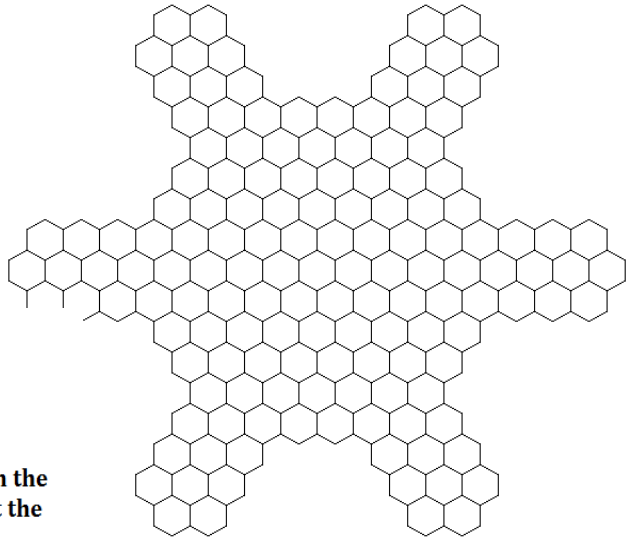


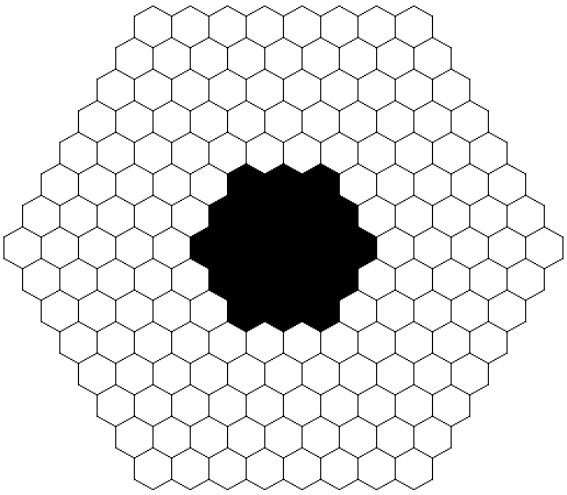
Pack the 10 x 15 parallelogram grid with the entire set such that the mono-, di-, tri- and tetrahexes neither touch one another nor the outside edge. It is possible!





The black area is a hole. Fill the shaded area with polyhexes 1 through 4.



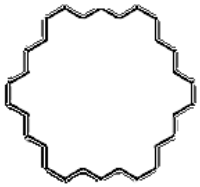


Pack the order-8 hexagonal grid with the entire set such that the mono-, di-, tri- and tetrahexes neither touch one another nor the edges of the figure - inside or out. It is possible!

HEX NUT

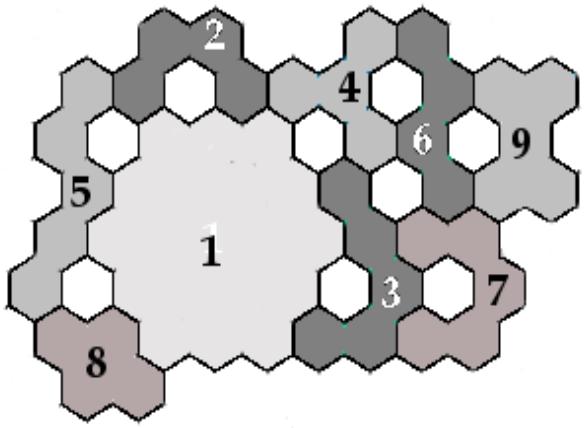
A strategy game for 2 to 4 players

Start: Place all tiles outside the play area as a common pool for players to draw from. The tray is not used. With 3 players, set the single hex aside, out of the game. With 4 players, set the single and dihex aside. Place the clear window piece as a starter tile in the center of the playing field. Have score sheet and pencil handy to record scores on every turn.



Play: Players take turns choosing a tile from the pool and adding it to any piece on the playing field, always touching at least 2 edges while trying to join as many edges as possible and forming as many single "hex" *holes* as possible. Score one point for each edge joined plus 6 points for each single empty space enclosed. Play until all tiles have been used up. Here are a few sample moves and their scores:

1. Starter piece.
2. 3 edge + 6 = 9 points
3. 5 edge + 6 = 11 points
4. 3 edge + 6 = 9 points
5. 4 edge + 6 = 10 points
6. 2 edge + 6 = 8 points
7. 6 edge + 6 = 12 points
8. 4 edge + 6 = 10 points
9. 3 edge + 6 = 9 points



Win: The player with the highest total point score wins.

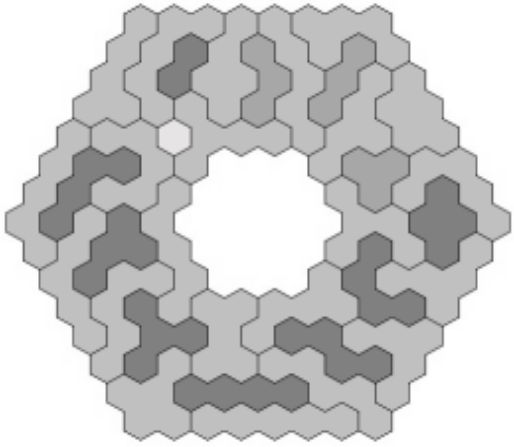
DOUBLE BUBBLE

A strategy game for 2 players

Start: Place the empty tray on a flat playing surface between the two players. Put the clear window piece into a corner of the tray. Set the monohex aside as it is not used for this game. One player takes all 22 pentahex pieces; the other player takes all the remaining 11 pieces.

Play: Take turns placing pieces into the tray, always touching at least two edges of another piece. The pentahex player goes first and always places *two* pieces per turn. Pentahexes must always connect. The other player always places one piece per turn. None of that player's pieces may touch each other nor the edge of the tray. Play continues until one player is not able to play or until all pieces are placed. That ends the first round. Each player gets a point for the number of pieces the other player has played. Clear the tray except for the clear window and reverse positions for the second round.

Winning: Overall winner is the player who gained the most points cumulatively in the two rounds, based on enabling the other player to play as many pieces as possible. If there is a tie, or if all pieces are played, both players win.



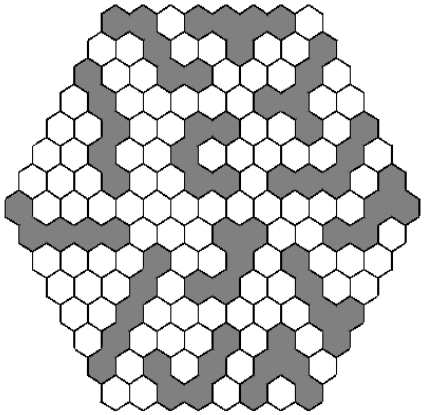
Because the game starts with the clear window in a corner, a symmetrical arrangement like this will not occur, though players will aim for a similar look and distribution.

PENTA PLAY

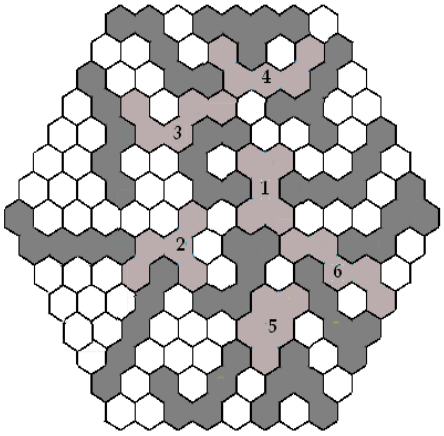
An enabling strategy game for 2 players

Start: Use only the pentahexes in a common pool. Place the grid between the players. First player chooses any pentahex and places it on the grid.

Phase 1: Take turns choosing a piece from the pool and placing it onto the grid so that no two pieces touch each other. The strategy is to *avoid* being the last player able to play, instead enabling the other player to have room for another piece. The player who can't fit another piece into the tray gets a point for each piece played. The example at right earns 13 points.



Phase 2: At this point, continue taking turns adding one new piece at a time, connecting together as many pieces as possible with each move. On each turn score one point for each piece in the connected group. At right, the order of play earned 4, 7, 10, 13, 17, and 19 points, respectively. Play until all pieces are either on the board or all pieces on the board are connected into a single network, which ends the game. The player who makes the full connection move gets one additional point for each piece, if any, not yet on the board. In the example at right, there are 3 pieces still not played, and so the player who made the completing move (the piece marked 6) earns 3 additional points. These bonus points are not awarded when the game ends without full connection.



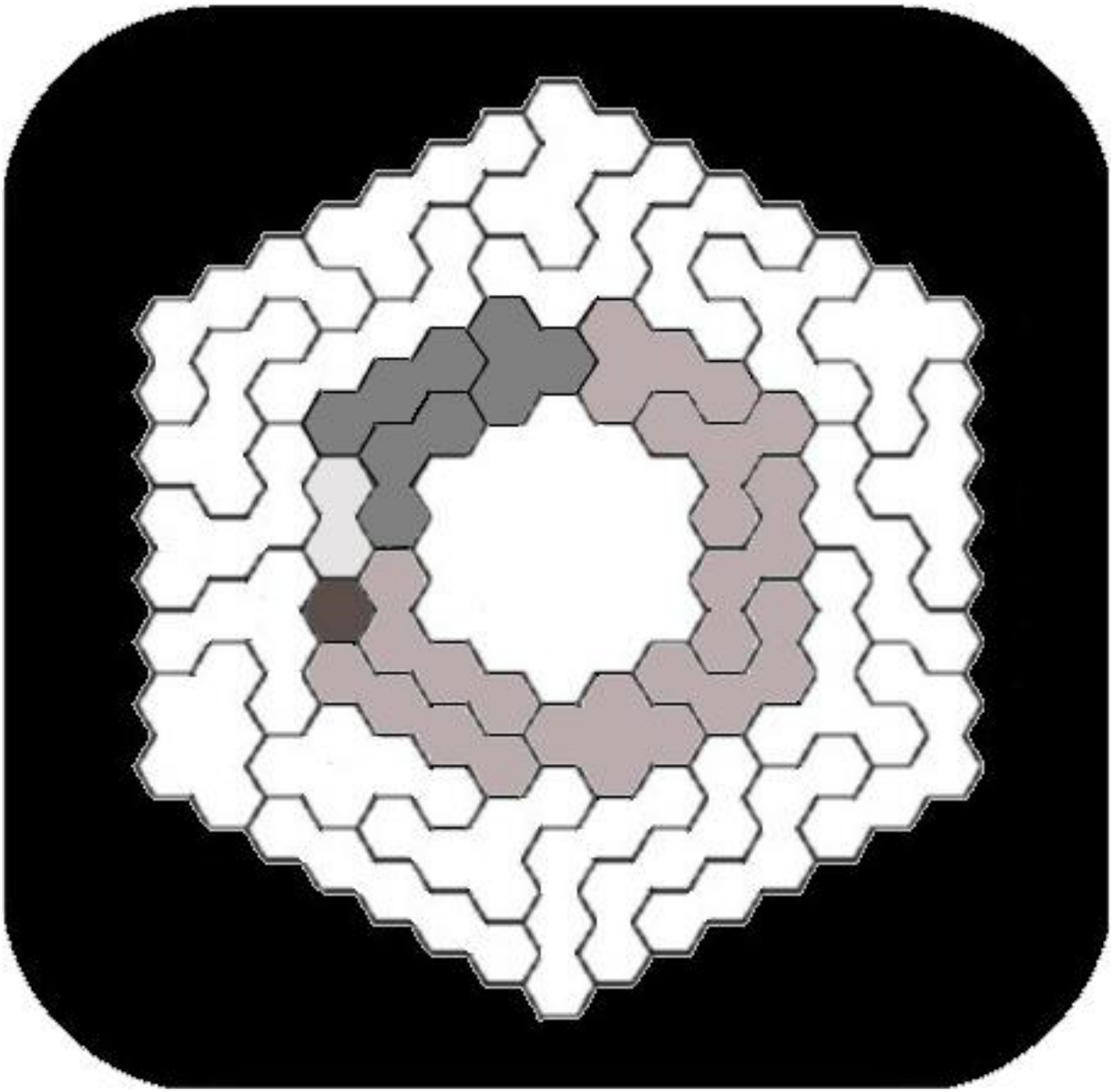
Winning: The player with the highest total point score wins. In case of a tie, if there are pieces not yet played, continue placing them if possible, scoring 1 point each. If there is still a tie, both players win.

ACKNOWLEDGMENTS

Polyhexes have a long and illustrious history, going back to at least 1961. Many scholars, puzzlers, authors, recreational mathematicians and hobbyists have explored them and made amazing discoveries, much of it unpublished or in private circulation. The Internet has made possible a far wider distribution of this wealth of information.

For the material in this booklet we have drawn upon the findings of many notable individuals whom we want to thank and acknowledge here. Please let us know if we've left anyone out. We regret that several of these eminent individuals have passed away. The following have contributed to polyhex research and to our own efforts: Elijah Allen, Sue Bare, Dave Barlow, Andris Cibulis, Andrew Clarke, Stewart Coffin, Joe DeVincentis, Peter Esser, Erich Friedman, Martin Gardner, Solomon Golomb, David Goodger, Michael Keller, Robin King, David Klarner, Dan Klarskov, Jürgen Köller, Joseph Lemire, Andy Liu, Jan Meeus, Thomas H. O'Beirne, Ed Pegg, Jr., Mike Reid, George Sicherman, Torsten Sillke, Pieter Torbijn, Miroslav Vicher, Eric Weisstein, Livio Zucca. Major online references:
http://abarothsworld.com/Puzzles/Polyhex/Polyhexes.htm (Dave Barlow)
http://puzzler.sourceforge.net/docs/polyhexes.html (David Goodger)
http://www.recmath.org/PolyPages/PolyPages/Polyhexes.html (Andrew Clarke)
http://www.vicher.cz/puzzle/polyforms.htm (Miroslav Vicher)

Polyhexes are just one member of an ever-growing family of mathematical sets known as polyforms—tiling puzzles based on combining multiples of the same basic shape, such as squares, triangles, rhombuses, pentagons, and many more, in every possible different shape at every level, from the singularity to as high as a researcher can count. And Kadon makes most of them. Computer-assisted searches go farther. The fun then is to see how many ways a selected group can form larger figures and patterns, finding solutions to questions that never end. It's even a branch of mathematics: combinatorics. The puzzle challenges in this booklet are just a sampling of what is possible. You, too, can solve amazing combinations and create surprising beauty; it just takes a little ingenuity and patience. ***Happy puzzling!***



Hexnut® *from Kadon*