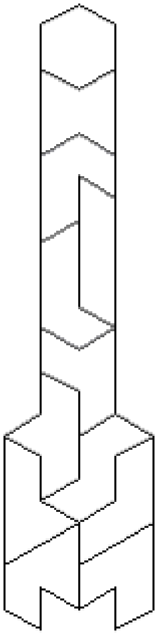
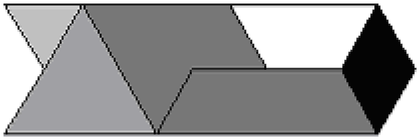
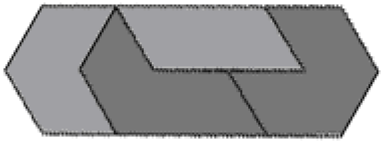
FOR 1 TO 6 PLAYERS
Ages 8 to adult



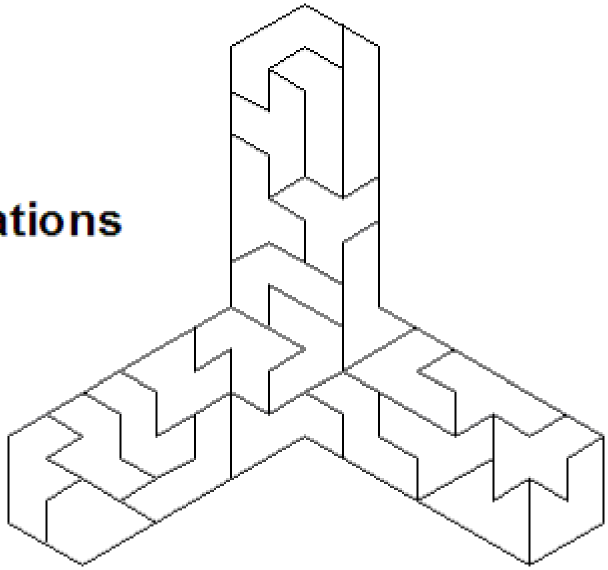




IAMOND RING™

The polyiamonds 1 through 7

- **Strategy Games**
- **Puzzle Combinations**



*A product of
Kadon Enterprises, Inc.*

IAMOND RING *is a trademark of Kadon Enterprises, Inc., for its set of 46 polyiamond game/puzzle pieces, representing the 46 possible combinations of 1 through 7 equilateral triangles joined at their edges. Each size has its own color.*

CONTENTS

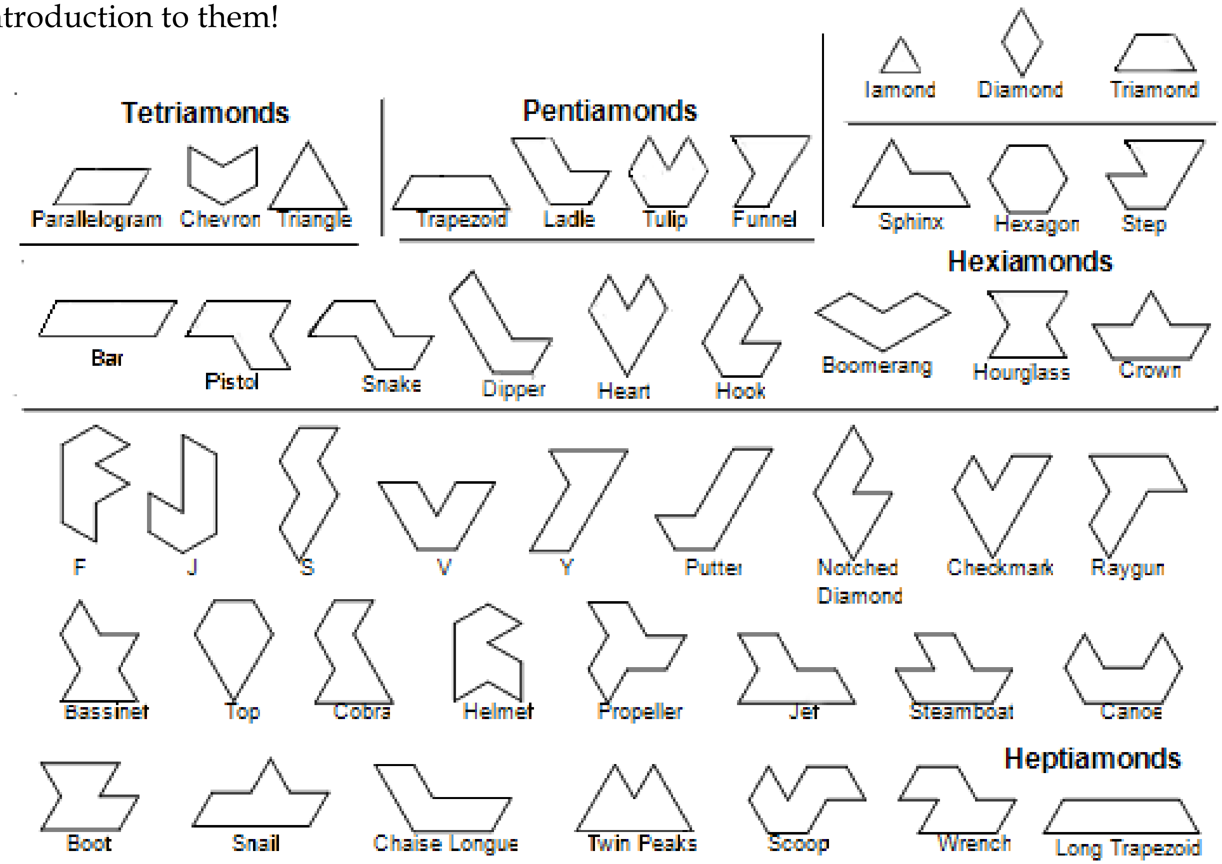
Games for two or more players

The beautiful acrylic game set was laser-cut and styled by Kadon Enterprises, Inc.
This book was compiled by Ed Pegg Jr. (www.mathpuzzle.com)

INTRODUCTION

The pieces in your IAMOND RING set are known as *polyiamonds*—the shapes you get when you join two or more equilateral triangles on their edges in every possible way. Here are the pieces, with the names we've given them to make them easier to recognize and remember.

Each size of polyiamond in your set has its own color. Many puzzles can be made by joining or separating the pieces by color. Purists will try to solve most of the puzzles in this book with the pieces of just one color. As starting examples, a perfect six-pointed star can be made with eight of the twelve hexiamonds, and a larger perfect six-pointed star can be made with the remaining four hexiamonds and all 24 heptiamonds. There are billions of puzzles to solve. This book is just an introduction to them!



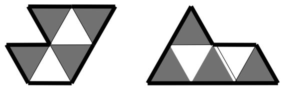
ABOUT POLYIAMONDS

THE IAMOND RING set is made up of the complete group of shapes known in recreational mathematics as "*polyiamonds*," a term coined by Thomas H. O'Beirne in the British periodical, *New Scientist*. He and Richard K. Guy solved a number of *hexiamond* problems in 1959.

The polyiamonds consist of: 1 iamond, 1 diamond, 1 triamond, 3 tetriamonds, 4 pentiamonds, 12 hexiamonds, and 24 heptiamonds. These are all included in your set. The *Octiamond Ring* consists of the 66 octiamonds and is also available from Kadon. There are 160 enneiamonds (figures made from 9 triangles). Can you find the unique enneiamond with a hole? Enumerations of higher orders can be found on the Web. Just search for *polyiamonds*.

The ten smallest pieces (the mini-iamonds) have a total area of 38 triangles. Kadon has a small book devoted to just these pieces. Highlights from this book are on page 8. One puzzle to try is to leave out the diamond and make a triangle with a side length of 6.

The twelve hexiamonds have a total area of 72 triangles. One might think that a triangle with a side of 6 could be made with them. Sadly, no. Here's an elegant proof. The individual triangles of these pieces can be colored like a checkerboard, as follows:



As it turns out, these are the only two hexiamonds that can be colored with two triangles of one color and four of another. This means that a figure made with the hexiamonds must have the same number of triangles of each color, or an excess of four triangles of one color. This is a powerful tool for determining whether hexiamond figures are possible. A triangle with a side of 6 has an excess of six triangles of one color, and thus cannot be made with six hexiamonds.

Solutions to any specific puzzle are available. Send a stamped, self-addressed envelope plus 50 cents to Kadon Enterprises, Inc., 1227 Lorene Drive, Suite 16, Pasadena MD 21122. Any questions? Phone (410) 437-2163, or email to kadon@gamepuzzles.com.

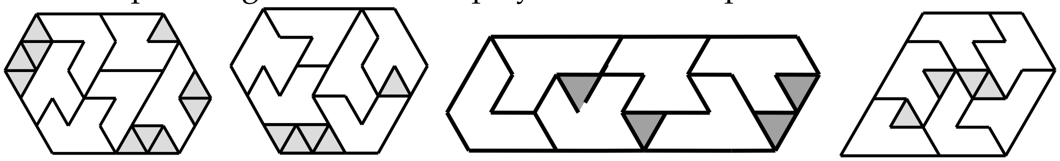
We wish you many happy adventures in pursuits and discoveries with the Iamond Ring.

Game One: “CONVEXITY”

Start: The 24 heptiamonds are evenly divided between 2-4 players. Whether rapid dealing, or selecting the pieces one by one, or just grabbing from a jumbled-up pile, follow this: use a method where the “good pieces” get divided evenly!

Play: Each person tries to fit their pieces together in the most convex shape possible (a shape with no indents). Zero is perfect. Twelve is the default highest value. As each person obtains a score, they announce it. Play continues until a majority calls “**Time**.”

Goal: Players receive a score of the number of triangles required to fill in holes and notches obtain convexity. The following figures have convexities of 10, 5, 4, and 4. These pieces can actually make a figure with a convexity of 1; can you find it? Each player wants the lowest score possible. After scoring everyone (the maximum score for one play is 12), divide the pieces again. When one player reaches 60 points, the lowest scorer wins.



Hints: A score of 7 or 8 can usually be found rather quickly. Low scores take time and skill.

Game Two: “VEX”

Play: Same as in CONVEXITY, plus “Vexing” is allowed. For the first game, a player may “Vex” the person on their right, once. Subsequent games allow a player to “Vex” the person on their left, once. A person may avoid being Vexed at any time by putting their hand over their figure and calling “**Freeze**,” but they may not adjust their figure after that, and therefore cannot Vex anyone. To Vex your appointed person, give them a piece you don’t want and point to a piece you do want while saying “**Vex**.” That person must give you the piece you indicated, and take the piece you gave them. Each person can Vex and be Vexed only once. A person who has called “Freeze” cannot Vex or be Vexed.

Goal: Same as in CONVEXITY.

Hints: Don’t Vex too early or too late. Good luck!

Game Three: “BOUNDARIES”

Start: Two, three, or four players divide up the 12 hexiamonds equally. The 24 heptiamonds are used as a common pool. The mini-iamonds are not used.

Play: Take turns placing two heptiamonds on the game board, choosing any pieces remaining in the common pool. Play continues until no more heptiamonds can be placed on the board. (Advanced game: place one heptiamond per turn.)

Once the above is completed, players begin to add their hexiamonds to the board, one each turn. *Hexiamonds may not touch the border, not even with a tip!*

Goal: To place the largest number of hexiamonds on the board.

Hints: During each turn, try to build enclosures that will accept only the one piece you are trying to stake out. Place blocking pieces in openings that are useful for the opponents. Try to keep track of how the endgame will play out.

Game Four: “STACK”

Start: Two players. Use heptiamonds to outline a (2,3) Hexagon (see figure on page 8) to serve as the gameboard. One player receives the iamond and pentiamonds. The other player receives the other 5 mini-iamonds. The 12 hexiamonds are in a common pool.

Play: Take turns placing a piece (one of yours, or one from the common pool) into the (2,3) Hexagon. The hexagon is assumed to be three (and *only* three) levels high. A piece may be played on the next level only if all the triangles under it are already occupied.

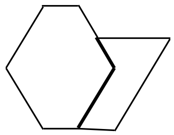
Goal: Last to play a piece wins. If all pieces are used, both players win.

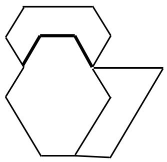
Hints: Try to force your opponent into using their pieces, while not using up your own. The hexiamonds are very strategic in this game.

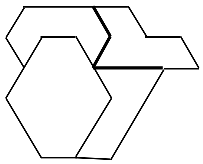
Game Five: "CONTACT"

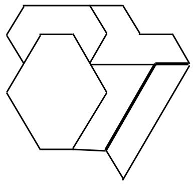
Start: 2, 3, 4, or 6 players. The clear center plug on a smooth, flat play surface. Divide the 24 heptiamonds equally among the players. If desired, the hexiamonds may be included.

Play: Take turn placing pieces on the table, joining them to each other and scoring points for unit edges brought into contact. The first piece is placed against the plug. Subsequent pieces may contact any piece or pieces. Score one point for each unit edge joined.


Three Points


Three Points


Four Points


Four Points

When all players' pieces have been joined in the central array, take turns relocating pieces, again scoring one point for every unit edge of contact. *Limits:* 1) Only a piece with at least one unit edge exposed at the outer perimeter of the array may be moved. 2) No piece may be moved that would disconnect another piece or leave it hanging by a corner. 3) No piece may be returned to exactly its previous position.

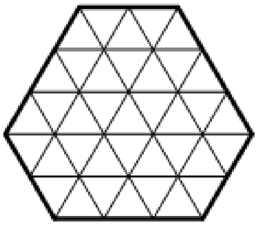
Take as many rounds of turns for the transformations as you did for the placement stage of the game. At their conclusion, remove pieces one by one, scoring one point for every unit edge *separated*. During this carrying-off stage, only pieces with an outer edge exposed may be taken, and no piece may be left disconnected from the central array. The plug is not carried off. The pieces you take become your "hand" for the next game.

Goal: To gain the highest score at the end of the game.

Hint: Try to set up spaces that will not score high for your opponent's pieces.

One optional rule for this game is to add the requirement that each piece placed must touch the last piece that was placed.

PUZZLES FOR THE MINI-IAMONDS

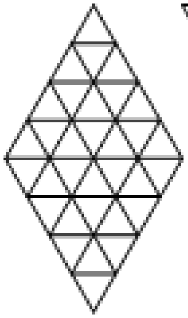


The figure on the left is a (2,3) Hexagon. Minus the single iamond, the nine remaining mini-iamonds can make this figure in 15,140 different ways. To start, just construct this figure; it is not too difficult. Then find solutions with one or more of the following conditions:

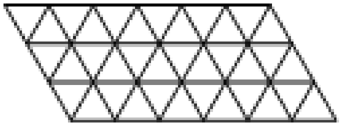
- Δ All the tetriamonds form a connected group.
- Δ All the pentiamonds form a connected group.
- Δ Three pieces do not touch the border.
- Δ No two of the tetriamonds touch each other.
- Δ Four pieces do not share a side with the border.
- Δ All the pieces are grouped by color.
- Δ Two pieces do not touch the border.
- Δ Every piece touches the border (hard).
- Δ No two pentiamonds touch each other.
- Δ Only two colors touch the border.

- Δ With the diamond starting anywhere, complete the figure. All positions are solvable.
- Δ Arrange the tetriamonds in a connected group so that they only touch the border with a tip.
- Δ Group all colors, but with the smaller colors only touching the pentiamonds.
- Δ The diamond and triamond do not touch each other nor the border, not even with tips.
- Δ Group all colors, but let the tetriamonds separate the pentiamonds from the other colors.
- Δ No two pieces of the same color touch each other. (This one is very tough!)

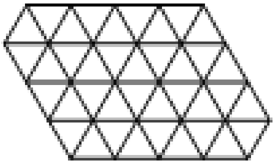
Here are other figures to solve with some or all of the 10 mini-iamonds:



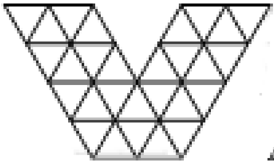
(4) Diamond



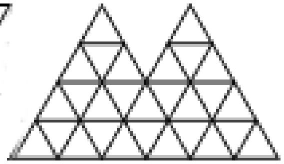
(3,6) Parallelogram



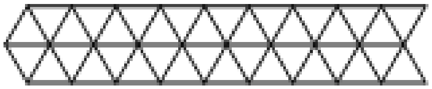
(1,4,3) Hexagon



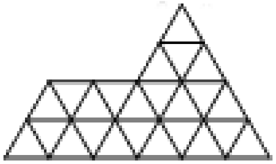
Doubled "V"



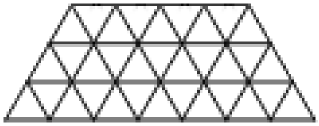
Doubled "Twin Peaks"



(9 long, 2 thick) Arrow



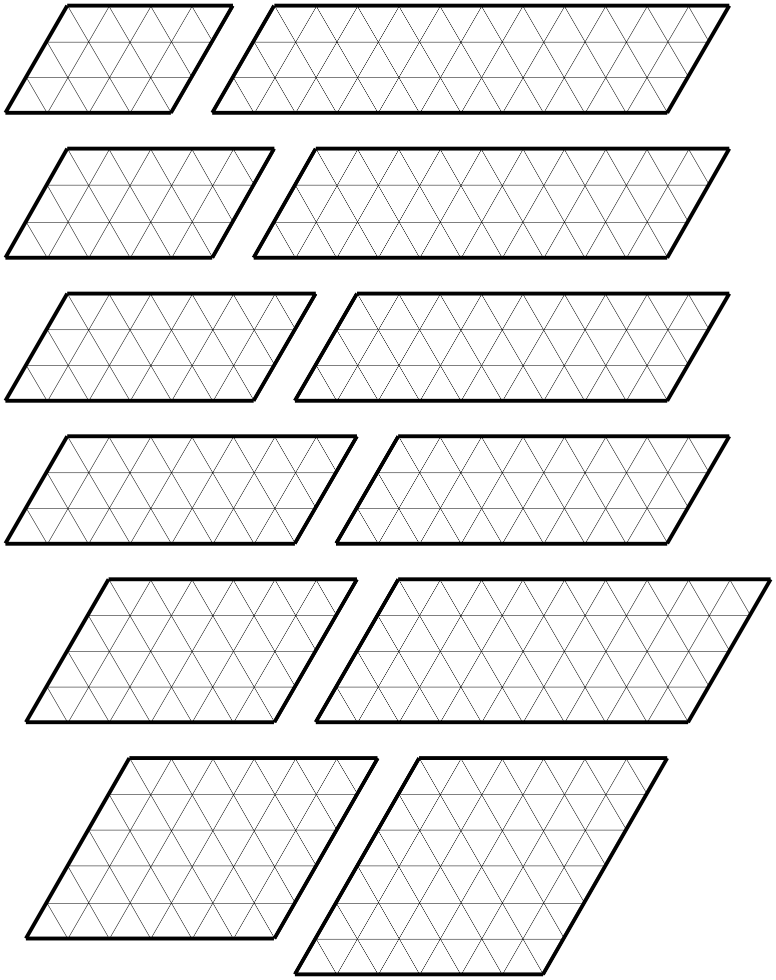
Doubled "Sphinx"



(3,4,3,7) Trapezoid

Can you make a side-6 Triangle with five hexiamonds and 2 mini-iamonds?

HEXIAMOND PARALLELOGRAMS AND TRAPEZOIDS

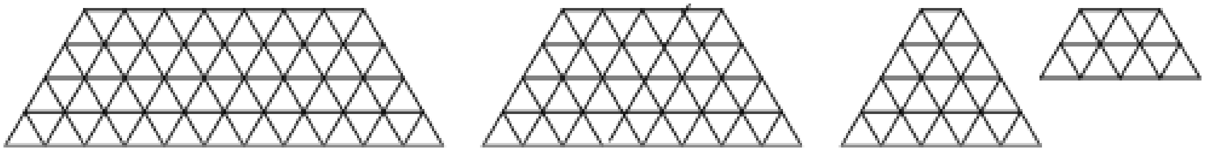


The first figures to make with the hexiamonds are parallelograms. The 2x6 parallelogram is also possible—try it; it's easy.

The 3x3 and 3x12 parallelograms are impossible. The 3x11 has 24 solutions, and all of them lack the Boomerang. The only parallelograms which use all 12 hexiamonds are the 4x9 (74 solutions), and the 6x6 (156 solutions).

Four different trapezoids are possible, shown below. You might enjoy demonstrating that other sizes of trapezoids are impossible by using a checkerboard coloring technique.

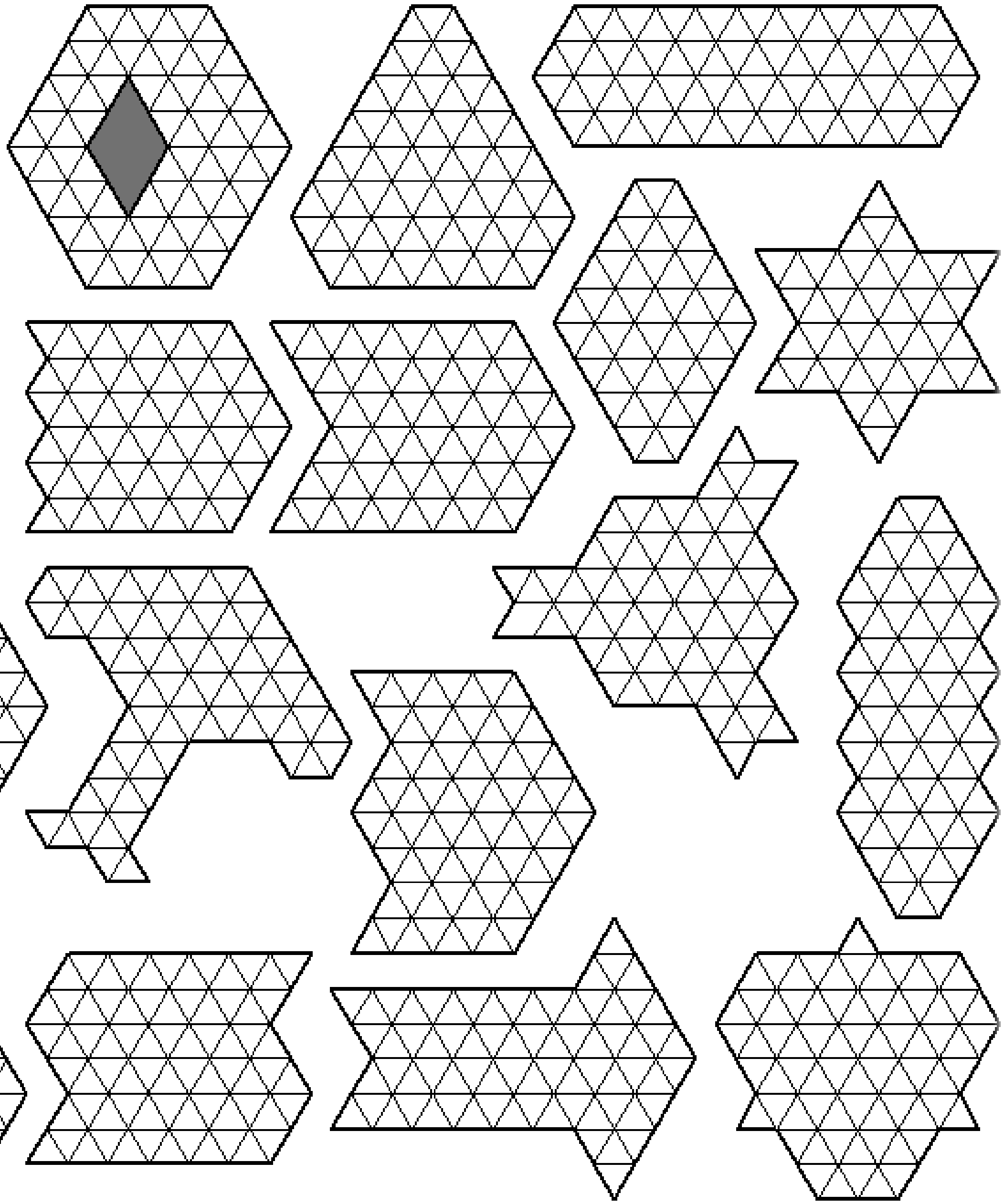
Try to find a solution for the 6x6 where the Bar does not touch the outer edge, and a solution to the 4x9 where there are no points where four or more pieces meet.

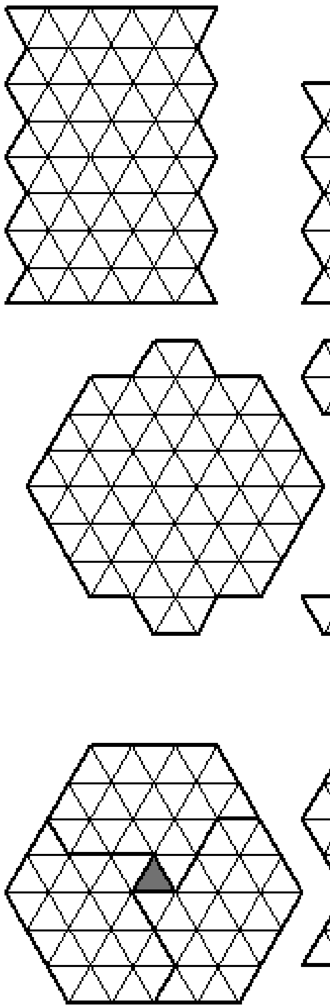


Another trapezoid has sides of (5,5,5,10). It has an area of 75 triangles, so there will be 3 holes. Can you build it with the 12 hexiamonds and the triamond?

HEXIAMOND PATTERNS

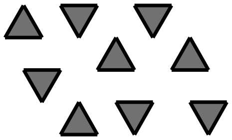
Many different patterns can be made with the hexiamonds. See how you do with these figures.

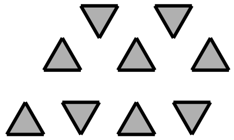




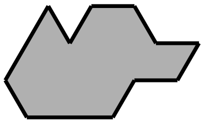
HEXIAMOND HOLES

Many interesting problems involve holes. Is it possible to arrange the 12 hexiamonds so that there are nine internal holes that do not touch either the perimeter or each other, even at a tip? The answer is yes – and here are two possible arrangements for *just the holes*:



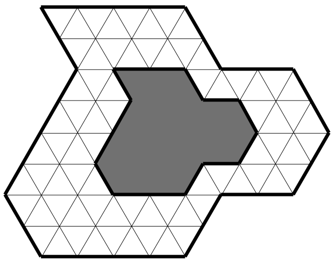


In the **Fence** problem, you use the 12 hexiamonds to make as large a hole as possible, with the hexiamonds forming a solid boundary around the hole, so that it doesn't touch the perimeter, even at a tip. The largest known fenced hole has an area of 114 unit triangles. The biggest known hole in a symmetric fenced figure has an area of 105 triangles. The longest twisting and turning hole in a fence we know of is 34 triangles long.

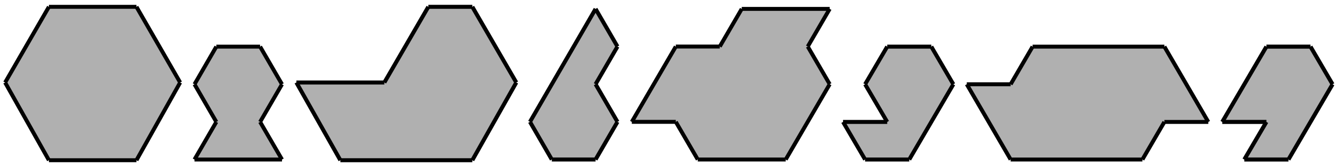


The hole to the left can be fenced in twice by separating the hexiamonds into two groups of six pieces. For the finest solution, make the perimeters of each fence the same shape as the hole. Yes, two identical figures can be made with a hole this big. (How large a hole can be placed within *three* identical figures?)

On page 9 concerning hexiamond trapezoids, you are asked to make a (5,5,5,10) trapezoid with twelve hexiamonds and the triamond. The triamond is a (1,1,1,2) trapezoid. If you pulled it out from the interior of the solution, the perimeter would be identical to the hole except for size.

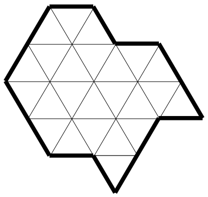


Here is a larger illustration of the **similar hole** problem. Examine the hole and the outer perimeter, and you will find that they are identical, except for scale. The outer edge doubles all the lengths found around the hole. This is a solvable figure. Other holes that have solutions for the "similar holes" problem are given below. The orientation and positioning of the outer perimeter has been left for you to find.

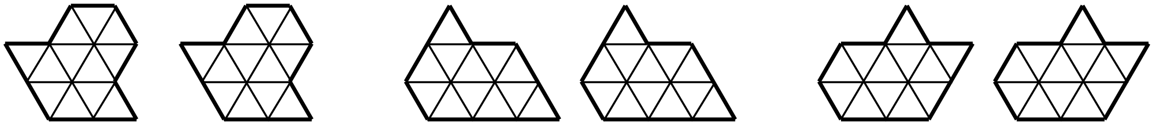


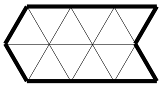
Each of the above holes can be surrounded by a larger but similar perimeter.

HEXIAMOND TASK PROBLEMS



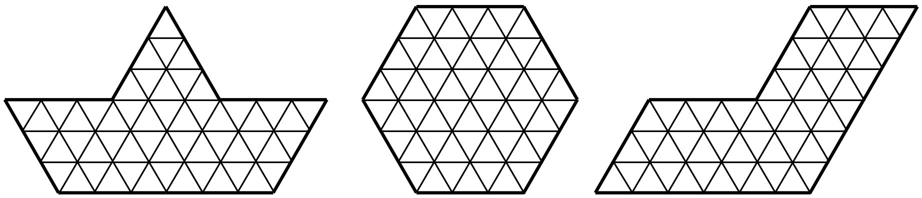
The figure to the left can be made with four hexiamonds. In fact, it can be made three times simultaneously. One way of abbreviating “Make the same figure 3 times using 4 pieces in each figure” is **(4 4 4)**. In this notation, you build figures of the same shape using the number of pieces specified in the parentheses. For (2 2) (2 2) (2 2) you would make three different pairs of identical figures, each with two hexiamonds. There are many different ways to do this. Another solvable task is (3 3) (3 3) – two pairs of identical figures, each with three pieces. Here is one example of (2 2) (2 2) (2 2) for you to solve:





(2 2 8) means “Make the same figure twice with two hexiamonds in each, then make a double-sized pattern with the eight remaining pieces.” The figure to the left is one of the patterns that can be solved in this way.

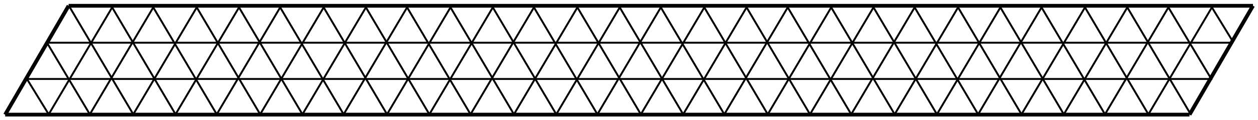
(1 9) is a short way of expressing the Triplication problem. It is impossible for the Steps, Sphinx, and Butterfly, but is solvable for all other hexiamonds. Set a hexiamond aside, and build the hexiamond three times as large with nine of the remaining pieces.



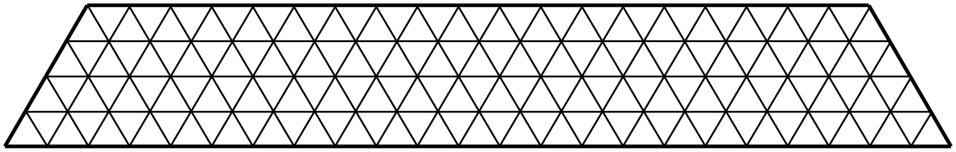
There are many different solutions to (4 4 4), (3 3) (3 3), (6 6), (2 2 8), (2 2 2) (3 3), and (1 9). Try to find new ones. Are the problems (3 3 3 3), (2 2 2) (2 2 2), or (2 2 2 2) (2 2) solvable? We haven’t seen a solution or a proof of impossibility for them.

One last task problem. Make a figure with 3 of the hexiamonds. Next, form the remaining 9 pieces into a pattern twice as large as the 3-piece figure. Again, this has many solutions.

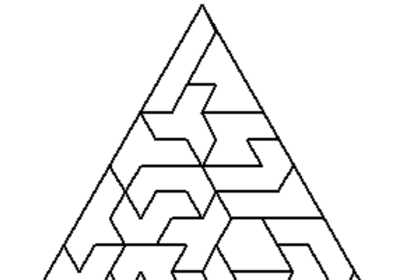
HEPTIAMOND PUZZLES



Several different parallelograms can be built with the 24 heptiamonds. For example, the 3x28 parallelogram above has many solutions. It uses all 24 pieces; 3x28**x2** gives the area in unit triangles (168). You can solve the 7x12, 4x21, and 6x14 sizes as well. It is even possible to make two 6x7 shapes simultaneously. All the following sizes are solvable with fewer than 24 heptiamonds: 7x2, 7x3, 7x4, 7x5, 7x6, 7x7, 7x8, 7x9, 7x10, 7x11, 3x21, 3x14, and 4x14.



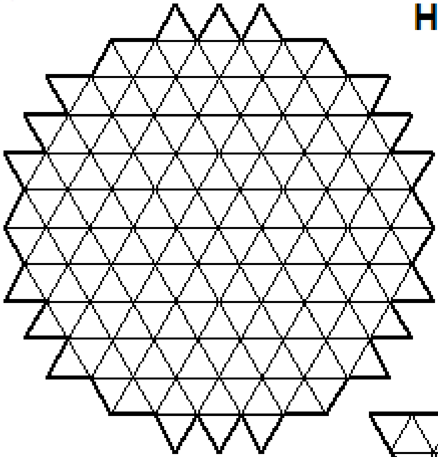
Several different trapezoids can also be constructed with the heptiamonds. The (4, 19, 4, 23) above is one solvable example. Two more that use all 24 pieces are the (12, 1, 12, 13) and the (6, 11, 6, 17). All of the following trapezoids are solvable with fewer than 24 heptiamonds: (7, 8, 7, 15), (3, 23, 3, 26), (7, 7, 7, 14), (10, 2, 10, 12), (7, 6, 7, 13), (7, 5, 7, 12), (4, 12, 4, 16), (8, 3, 8, 11), (7, 4, 7, 11), (5, 8, 5, 13), (3, 16, 3, 19), (7, 3, 7, 10), (6, 4, 6, 10), (7, 2, 7, 9), (7, 1, 7, 8), (3, 9, 3, 12), (2, 13, 2, 15), and (4, 5, 4, 9). Three others, all small, are for you to find. Is it possible to build several simultaneous trapezoids?

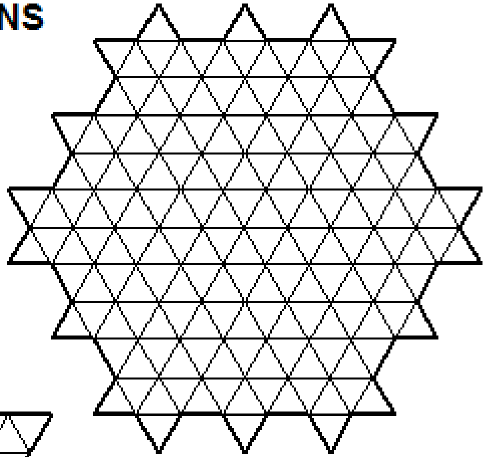


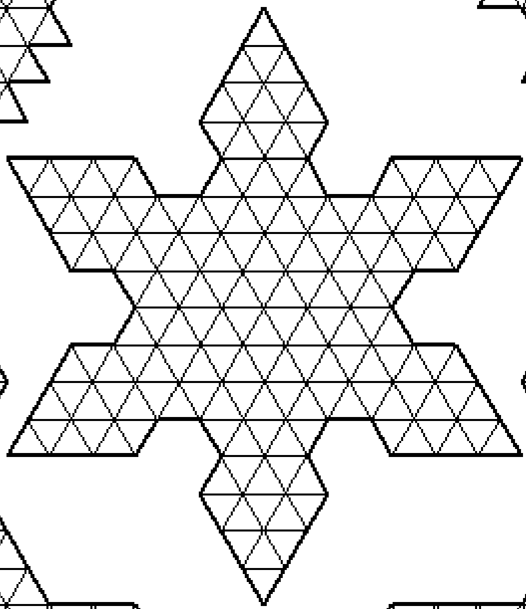
The triangle at left can be built with a single hole. It has a side of 13, so the total area in unit triangles is 13x13 = 169. The 24 heptiamonds have an area of 168 unit triangles, hence the hole. We’ve shown one solution as an incentive for you to find several other solutions, with the hole in different places. See if you can move the hole to the center.

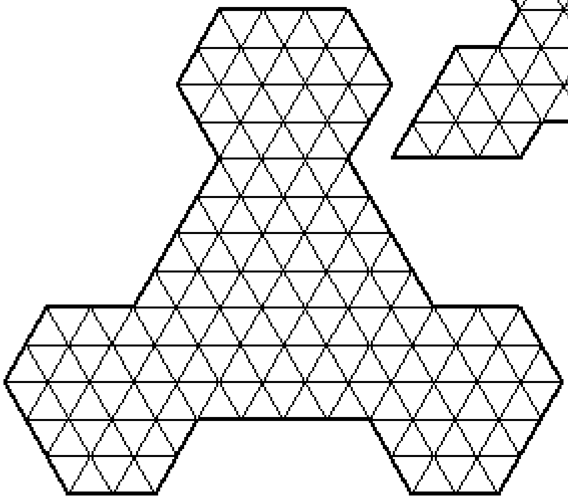
A perfect triangle with a side of 7 can be built with 7 pieces. It’s been proven impossible to build three of them simultaneously.

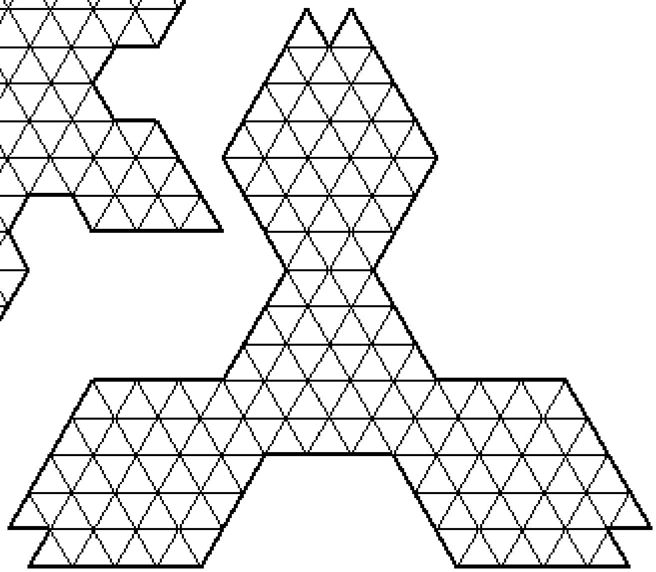
HEPTIAMOND PATTERNS



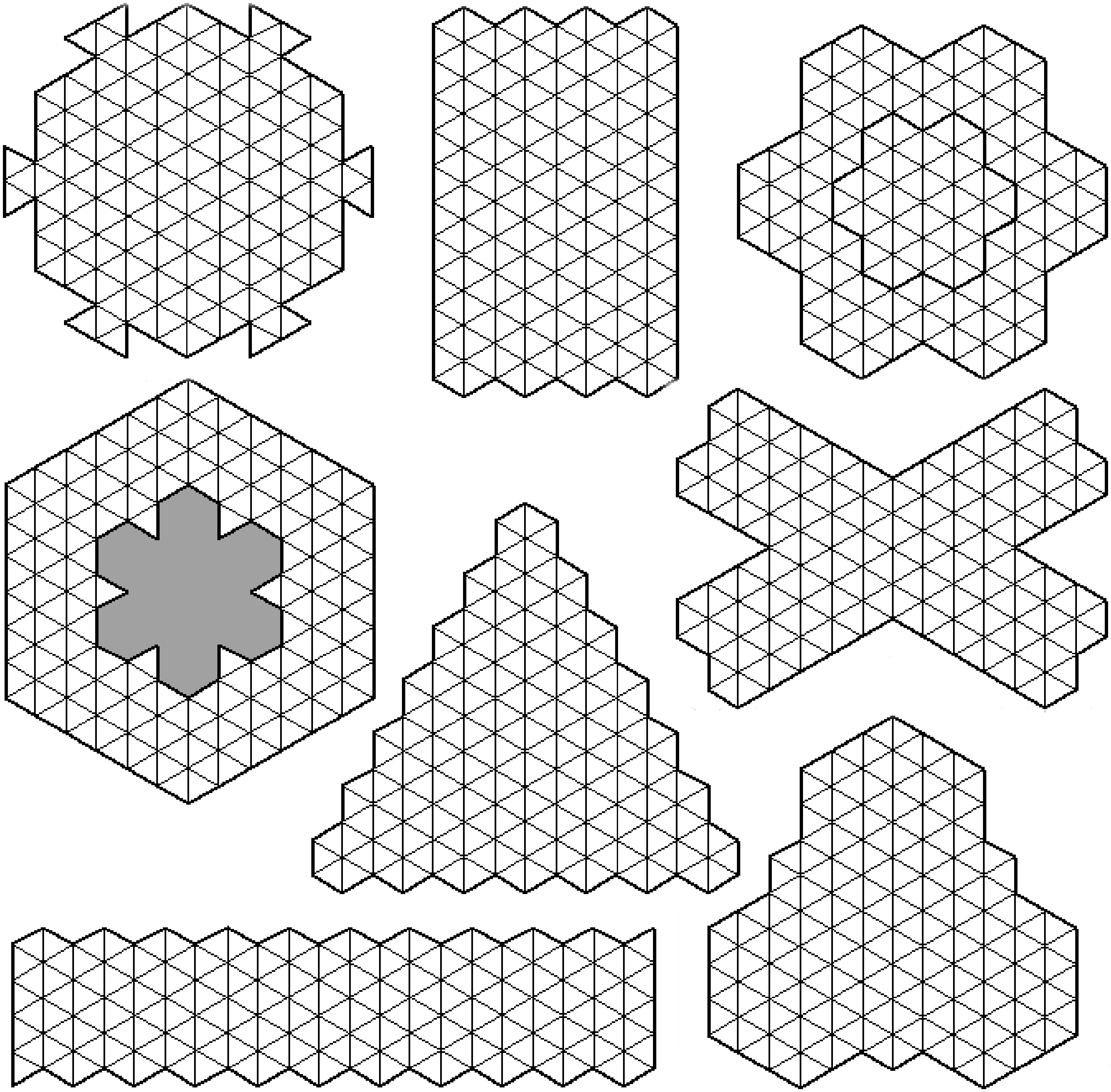




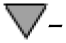


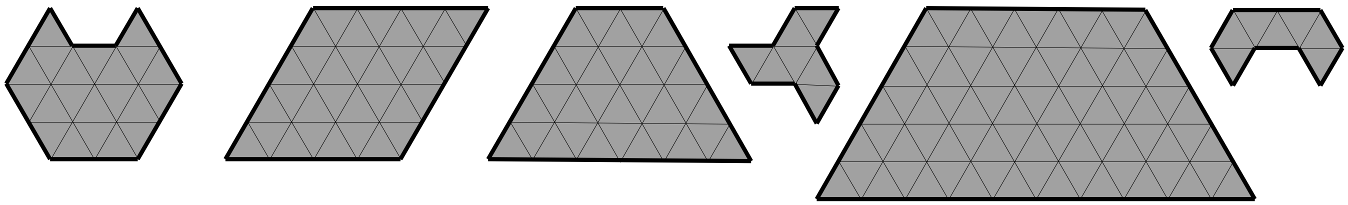


All of the patterns shown on these two pages can be solved using all 24 heptiamonds. If you color the heptiamonds with a checkerboard pattern, you will find that 23 of them have one extra triangle of one color, and Twin Peaks has three extra triangles of one color. For a pattern to be impossible due to coloring, it would need to have 27 extra triangles of one color. Can you find a figure which is impossible to make—for this reason, or some other reason? Many other patterns can be made with the heptiamonds. Experiment with variations on the figures here.

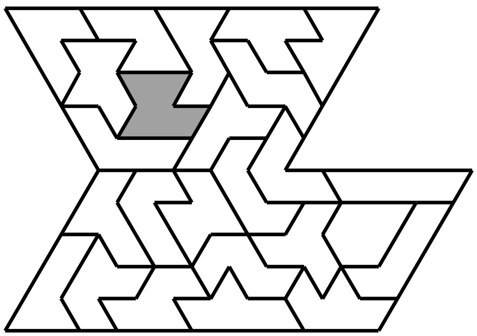


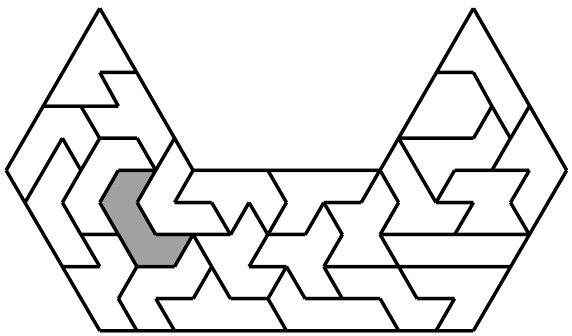
HEPTIAMOND HOLE PROBLEMS

This hole –  – can be surrounded by an identical figure 13 times as large (a side-13 triangle) using all 24 heptiamonds. Other “similar hole” problems solvable with the heptiamond set are shown below. In each, the outer perimeter will be the same shape as the interior hole.



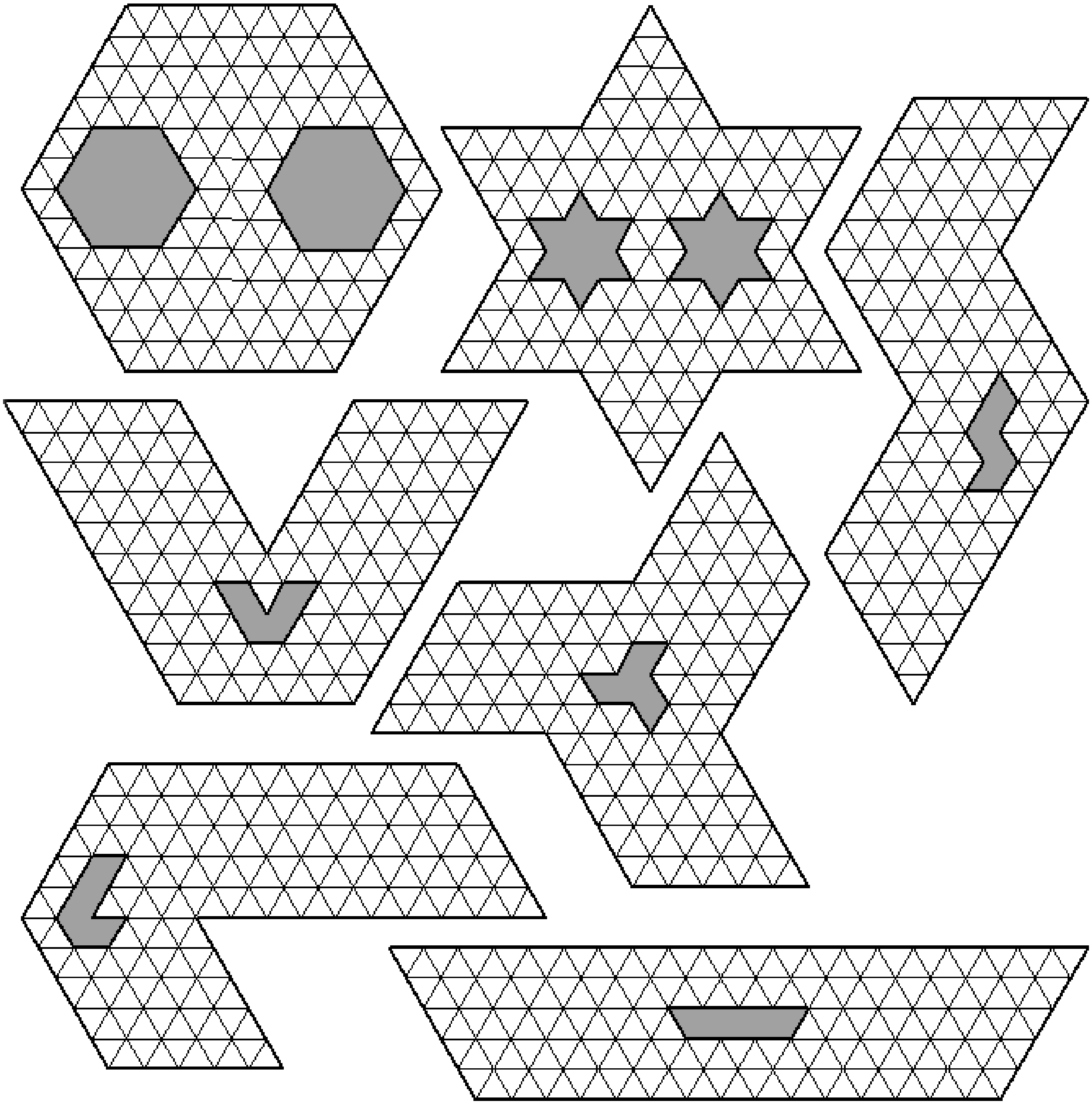
Two of the possible holes shown are themselves heptiamonds. Each heptiamond can be solved as a “similar hole” problem. In many, such as the Canoe, the hole can be both centered and oriented like the outer perimeter. Here are sample solutions. Try to fix the hole in the Canoe!



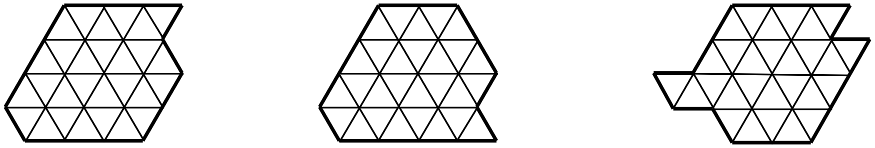


Double-hole solutions also exist. Make a six-pointed star with two interior holes that are also six-pointed stars. A hexagon with the same property is also possible. (See top of page 17.)

Many hole problems are unsolved for the heptiamonds. How many non-touching holes are possible? If the 24 heptiamonds are divided equally, what is the largest hole possible in 2, 3, or 4 identical figures? What is the maximum number of “up” pointing triangular holes possible in a figure? What is the largest hole inside a figure that is inside another hole (a fenced fence)? If you solve these, try to solve them again for symmetrical figures, and send us your answers!

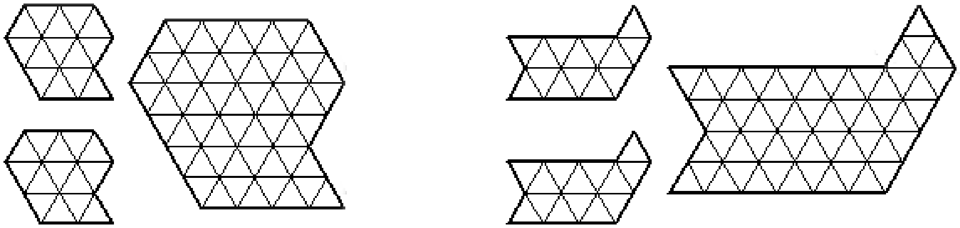


HEPTIAMOND TASK PROBLEMS

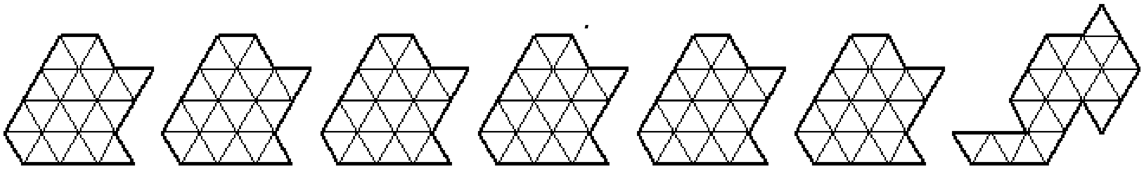


The three figures above each have the amazing ability to be sextuplicated with the full set of heptiamonds. Once you have six copies of one of these figures, make symmetrical patterns with them. We'll use the notation from Hexiamond Task Problems. Each figure above provides a solution for (4 4 4 4 4 4). Are there others? Here is a partial list of solvable task problems:

(2 2) (2 2) (2 2) (2 2) (2 2) (2 2)	(3 3) (3 3 3 3 3 3)	(2 2 8) (2 2 8)
(2 2) (2 2) (2 2) (2 2) (2 2 2 2)	(2 2) (4 4 4 4 4)	(2 2 2 2) (8 8)
(2 2) (2 2) (2 2) (2 2 2) (2 2 2)	(3 3 3 3) (4 4 4)	(3 3 3) (3 12)
(2 2 2) (2 2 2) (2 2 2) (3 3)	(3 3 3) (5 5 5)	(1 4 9) (2 8)
(2 2 2 2) (2 2 2) (2 2) (3 3)	(4 4 4) (4 4 4)	(2 2 2 18)
(2 2) (2 2) (2 2) (3 3 3 3)	(2 2 2) (6 6 6)	(6 6 6 6)
(2 2) (2 2) (2 2) (2 2 8)	(2 2) (5 5 5 5)	(4 4 16)
(2 2 2) (3 3) (3 3 3 3)	(2 2) (2 2 8 8)	(8 8 8)



Outline of a solution for (2 2 8) (2 2 8)



Outline of a solution for (3 3 3 3 3 3) (3 3). The last figure should be duplicated.

Many of these can be broken down into simpler problems. Many harder problems remain unsolved. For example, is (3 3 3 3) (3 3 3 3) possible? Let us know and win a prize.

HYBRID AND FULL-SET CHALLENGES

- The whole set has an area of 278 (=2 x 139). One interesting task that can be solved with the whole set is to make 7 simultaneous triangles. Triangles with sides of 1, 3, 3, 4, 5, 7, and 13 can be made with the full set. Is a different set of 7 triangles possible?
- If you remove the diamond, a 6x23 parallelogram can be made, or a six-pointed star with a hexagonal hole of side 2. A trapezoid with sides (6, 20, 6, 26) is also possible.
- With the pentiamonds, hexiamonds, and heptiamonds, an 8x17 parallelogram can be made. Is it possible to build it so that the smaller pieces are separated?
- With the pentiamonds and hexiamonds, you can build four identical figures, each containing one pentiamond and three hexiamonds.
- A side-6 hexagon can be made with 8 hexiamonds and 24 heptiamonds, and a side-2 hexagon can be made with the remaining 4 hexiamonds.
- In the game **STACK**, pieces may not be added to the next level unless all the triangles underneath that piece are filled. Using just the hexiamonds, a tower with 6 levels is possible. Using just the heptiamonds, a tower with 9 levels is possible. How many levels are possible with the full set? If a full-set tower has 12 levels, how large can the top level be?
- Can a side-17 triangle with a side-7 triangular hole be built with the hexiamonds and heptiamonds? How about a side-16 triangle with a side-4 triangular hole?
- If 2, 3, or 4 people are playing **VEX**, can all tie with a score of zero? When six people want to play, the hexiamonds can be added to the distribution. Can six people tie?

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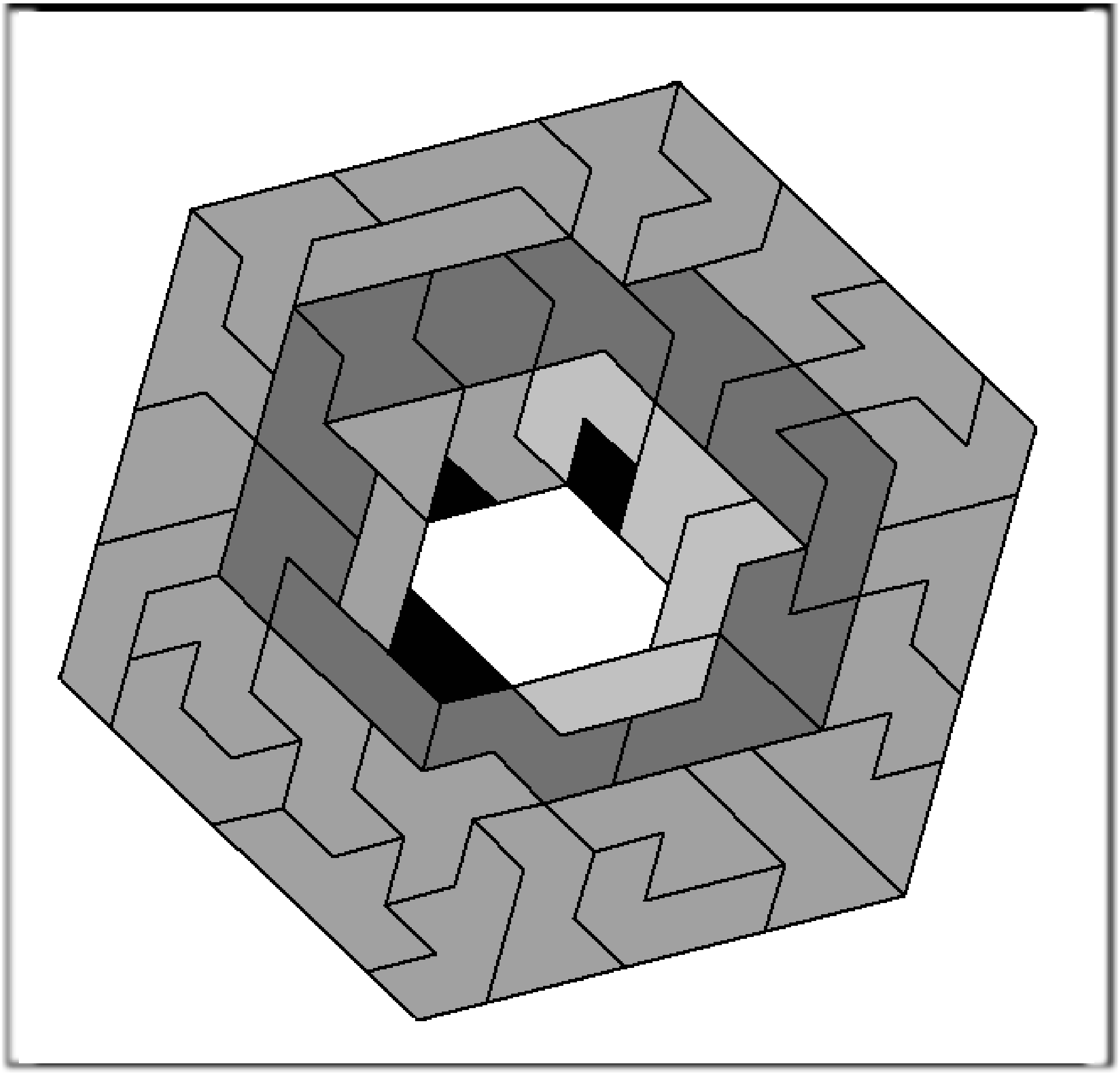

Iamond Ring™ *from Kadon*