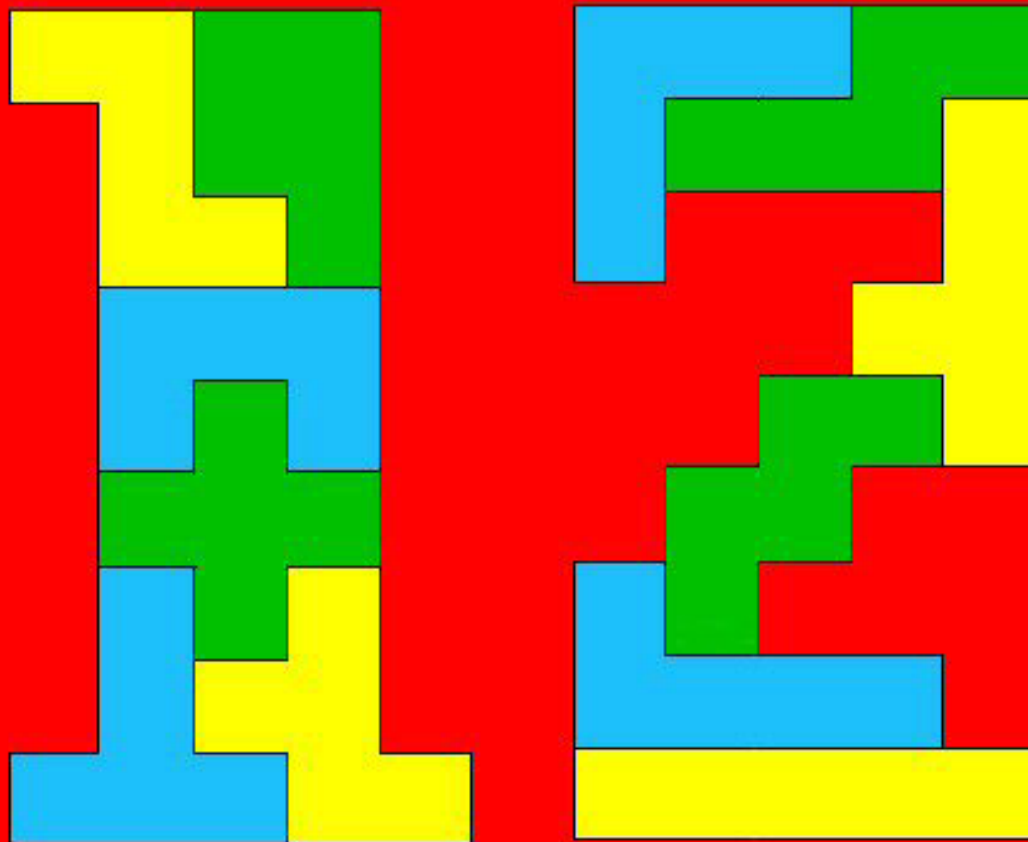


# A Fibonacci verse



**Bridges 2016—Jyväskylä, Finland**

# Twelveness

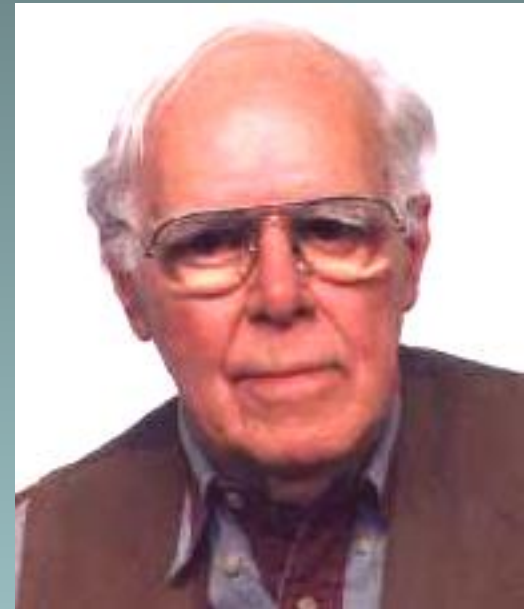
*A Fibonacci verse  
celebrating the 12 pentominoes*

**1 1 2 3 5 8 13 21 34 55 89 144**

Presentation by Kate Jones  
at Gathering4Gardner 12 – Atlanta, GA  
and Bridges 2016 – Jyvaskyla, Finland

**1 Martin**

**1 Gardner**



**2 long ago**

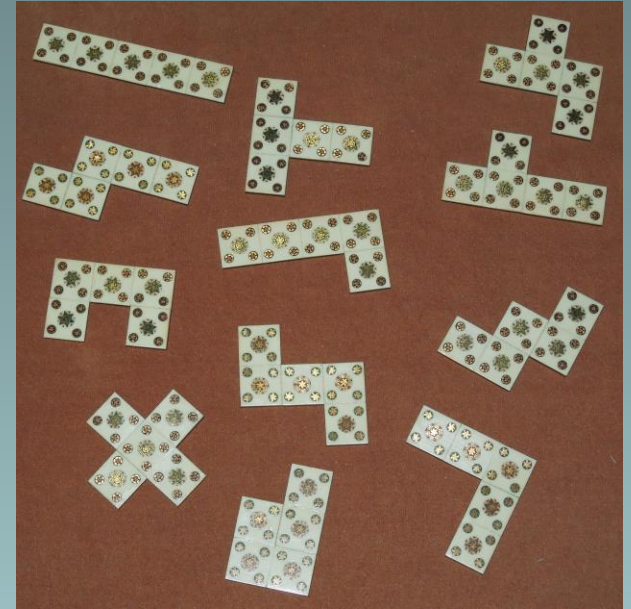
SCIENTIFIC AMERICAN

**May 1957**

Volume 196, Issue 5

**3**

**Wrote about  
pentominoes**



**5**

**Brainchild of young  
Solomon Golomb**



# 8 The coolest recmath set in all the world

## THE AMERICAN MATHEMATICAL MONTHLY

THE OFFICIAL JOURNAL OF  
THE MATHEMATICAL ASSOCIATION OF AMERICA, INC.

VOLUME 61



NUMBER 10

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DECEMBER

1954

*For Kate Jones,*  
*10/17/2012*  
*-Sol Golomb*

### CHECKER BOARDS AND POLYOMINOES

S. W. GOLOMB, Harvard University

Our starting point is the well-known problem: Given a checker board with a pair of opposite corners deleted (Fig. 1), and given a box of dominoes, where each domino covers exactly two squares of the checker board, is it possible to cover this checker board exactly with dominoes? The answer is "no"; for suppose that the checker board is colored in the usual manner (Fig. 1). Then each domino covers one light square and one dark square. Thus  $n$  dominoes would cover  $n$  light squares and  $n$  dark squares, that is, an equal number of each. But the checker board of Fig. 1 has more dark squares than light squares, and so it can not be covered with dominoes.

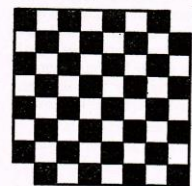


Fig. 1

We shall retain the  $8 \times 8$  checker board as our "canonical domain," but we shall generalize the "domino" to the "polyomino," and our theorems will involve all the simpler polyominoes, shown in Figure 2. More precisely, we define an  $n$ -omino as a simply-connected set of  $n$  squares of the checker board which are "rook-wise connected"; that is, a rook placed at any square of the  $n$ -omino must be able to get to any other square, in a finite number of moves.

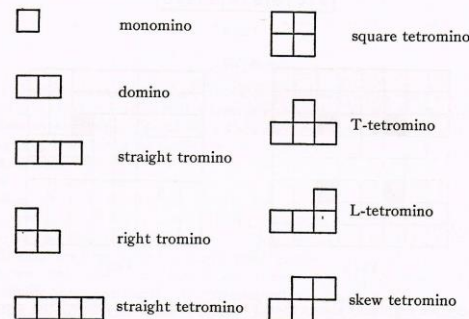


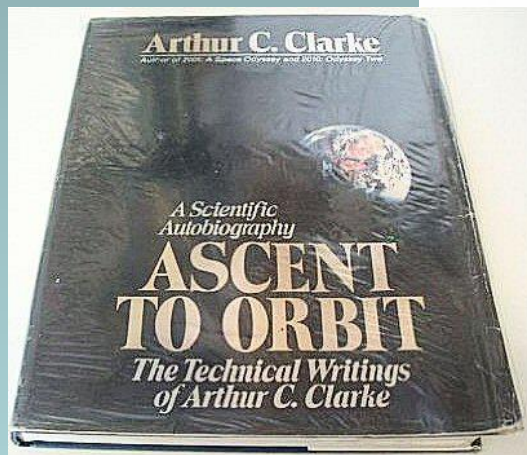
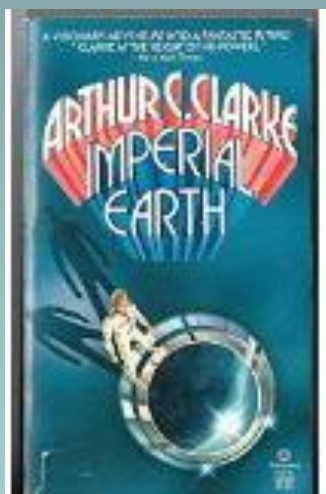
Fig. 2

First we consider trominoes. Clearly it is impossible to cover the  $8 \times 8$



# 13

Soon everybody played them, Gabriel made them, Even Arthur Clarke became their addict.



And in 1980 Kadon Enterprises (1227 Lorene Drive, Pasadena, Md. 21122—did you know that there was a Pasadena in Maryland?) sent me one of their beautiful 3-D sets, trademarked “Quintillions.” These are cut from extremely hard wood by a laser, so precisely that they fit together almost like machined metal blocks.

## 23 *HELP! I Am a Pentomino Addict!*



I cannot remember mathematics, but I comment when I first be the mind by revealing

The occasion was one of the rather rare lectures by our Headmaster at Huish's, Arnold Goodliffe. He was a large, imposing figure in the great tradition of English pedagogues; we all regarded him with awe and respect, but not with fear (unless there was good reason; in those far-off days corporal punishment was still permitted, and the Old Man knew how to apply it). His duties as Head must have left him little time for taking classes, but he must have been an inspiring teacher or he would not have made such an impact on my mind.

The lesson was one in elementary algebra. We all knew the simple formula for the sum of the first  $n$  integers:

$$n(n + 1)/2$$

and Arnold Goodliffe posed the question, which had probably never occurred to any of us: Is there a formula that gives the sum of the *squares* of the first  $n$  integers? He then proceeded to derive it by the process of induction, and I was at once struck by the power and elegance of the method. It impressed me enormously; but I had a bigger surprise in store.

When Dr. Goodliffe had written down the plausible but not very exciting solution

$$n(2n + 1)(n + 1)/6$$

# 21

Through a feat of fate along came Kate and started a business, because she could, founded on 12 pieces of wood.

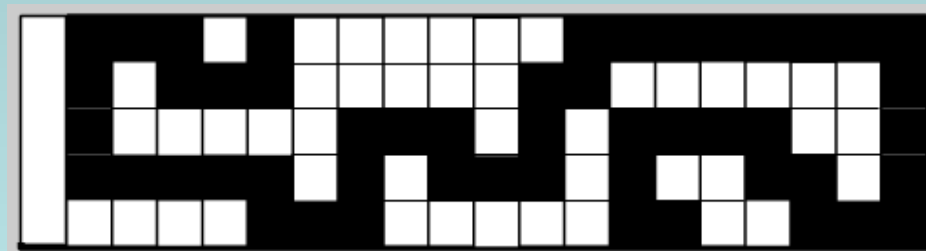
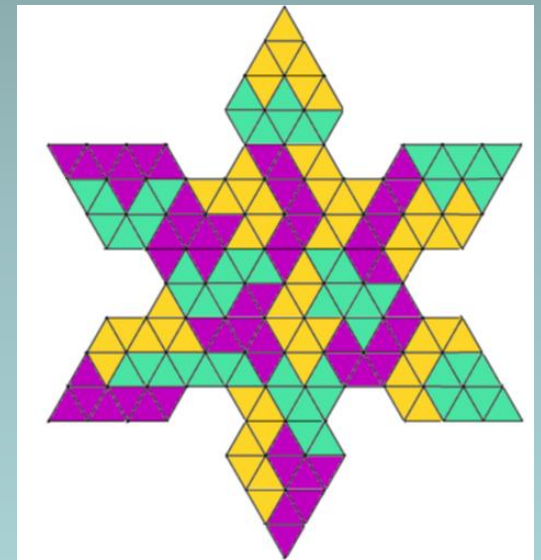
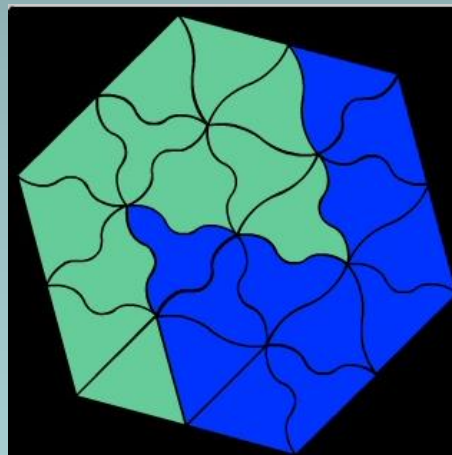
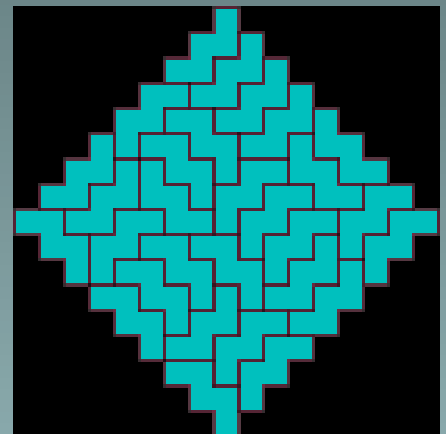
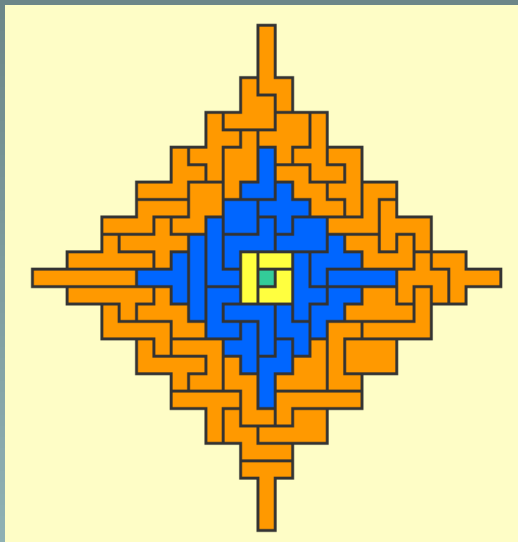


# 34

And this one set begat lots more—  
combinatorial puzzles by the score—  
as awards rolled in and ribbons flew  
and a beautiful mathematical product line grew,  
lovingly crafted... sold only in our traveling store.







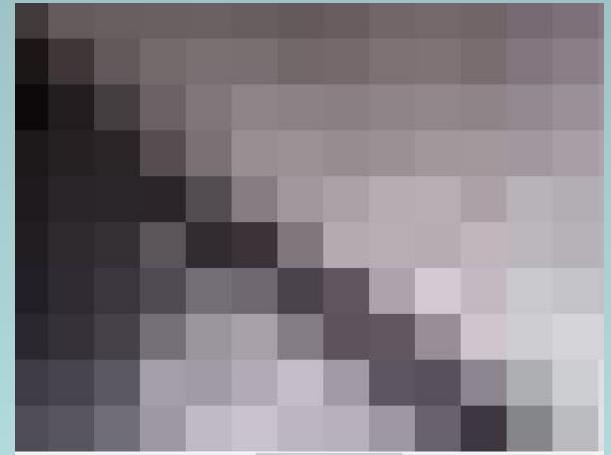
# 55

As decades flowed by, the pents we'd named Quintillions  
Stood ever in first place, and their fans grew by the millions.  
Their shapes showed up in a whole parade  
Of other creations that we made.  
And dear Martin Gardner, friend and mentor,  
Let us make the two games of which **he** was the inventor.



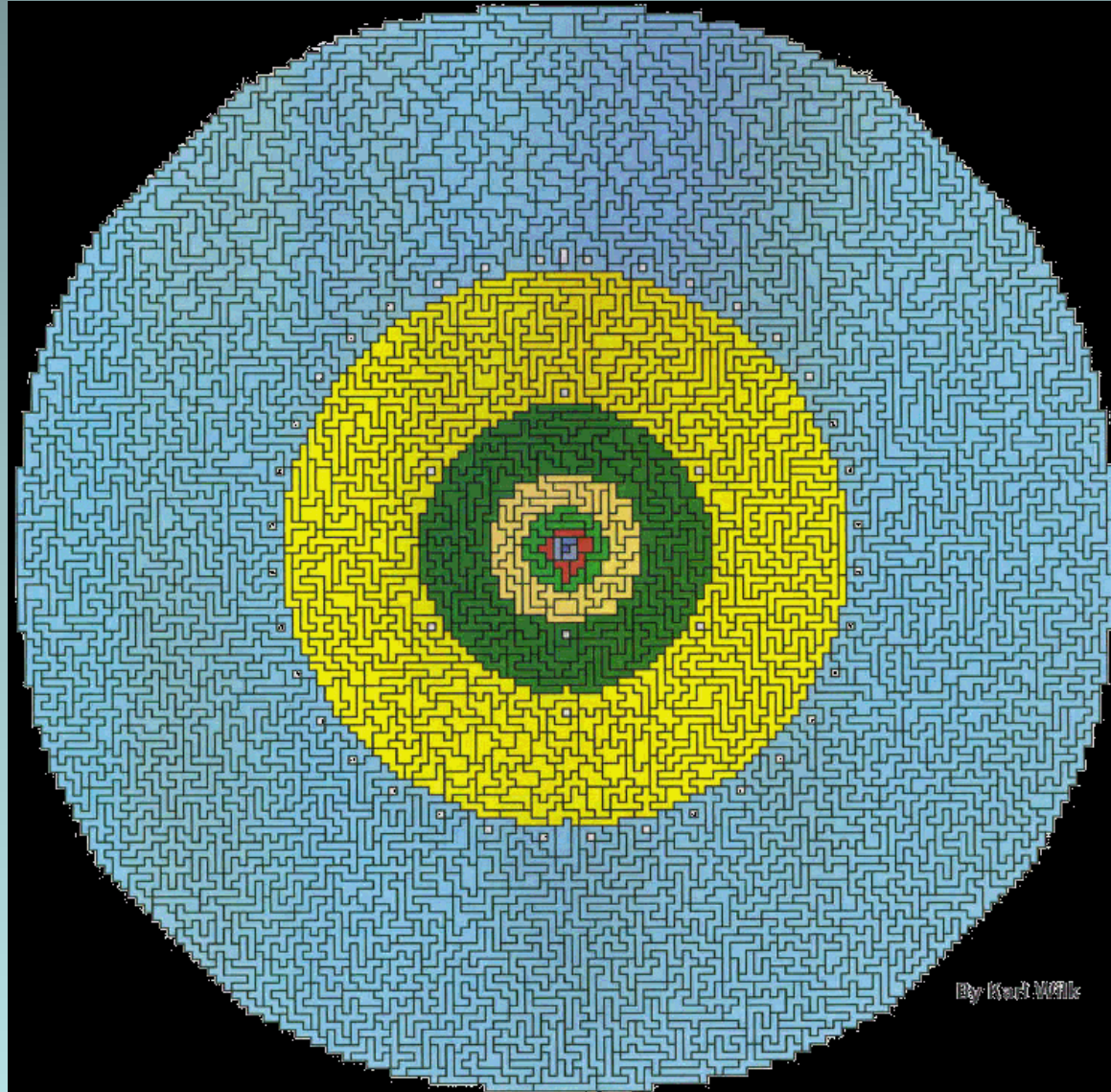
# 89

Polyominoes are everywhere, just take a look around —  
On floors and walls, on every web page as pixels they are found...  
From the Singularity to infinity, particles join in ever more fanciful arrays  
Like elements in galaxies, where energy with space-time plays;  
Then living beings happened along, from single cell to the giant whale  
And played with variations, inventiveness at every scale,  
And somewhere in the middle are these humans on a planet blue —  
They have minds that play with puzzles, math and the magic they can do.





From the Singularity to infinity, particles join in ever more fanciful arrays  
Like elements in galaxies, where energy with space-time plays;

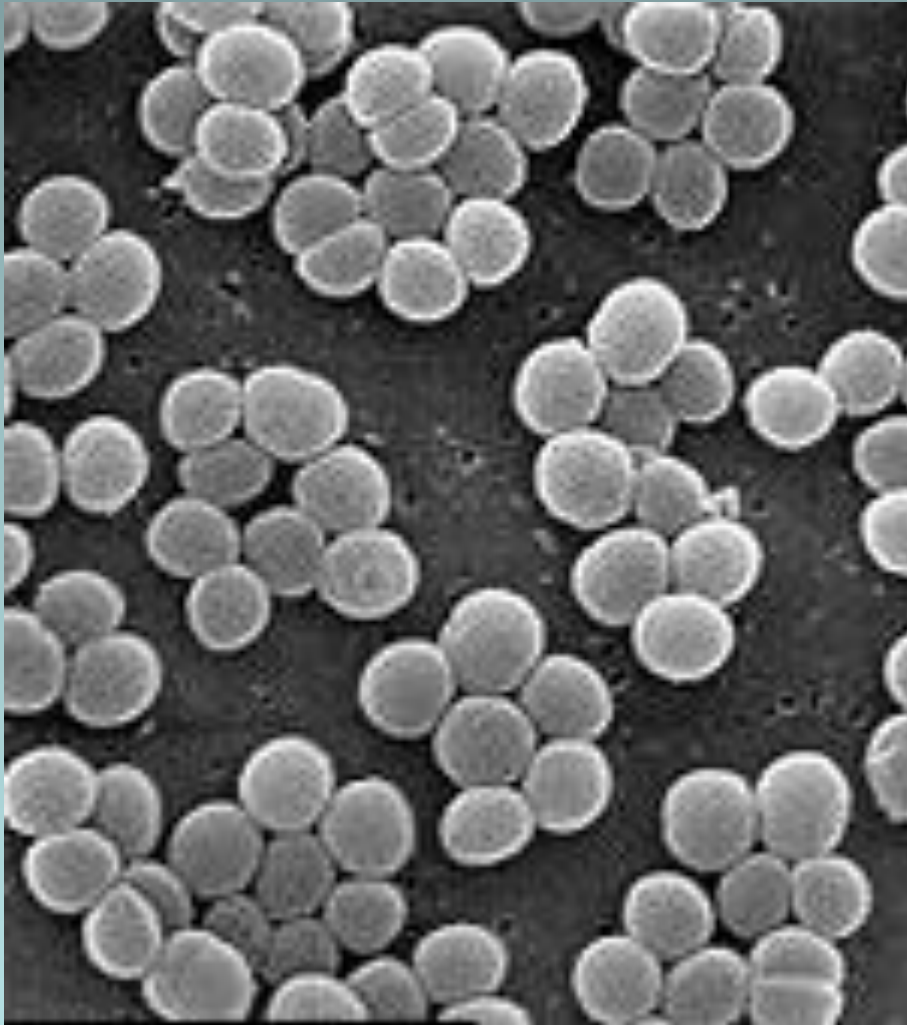


By Karl Wolk



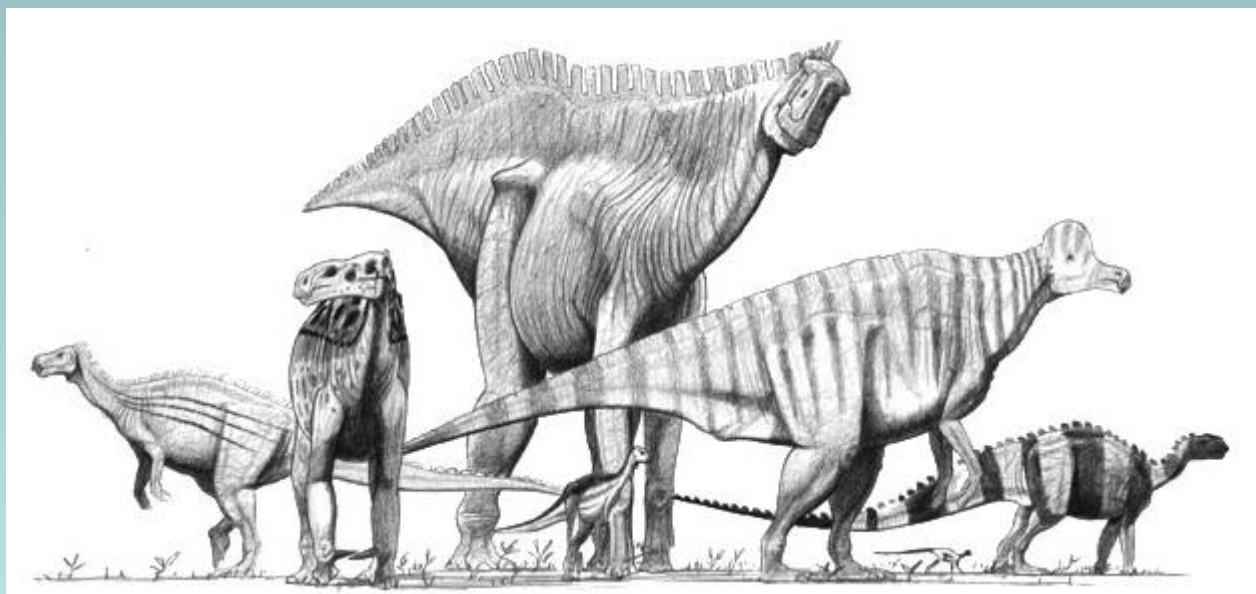


Then living beings happened along, from single cells to the giant whale  
And played with variations, inventiveness at every scale,



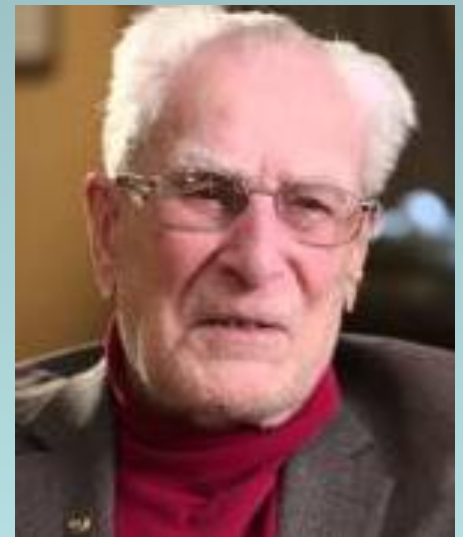
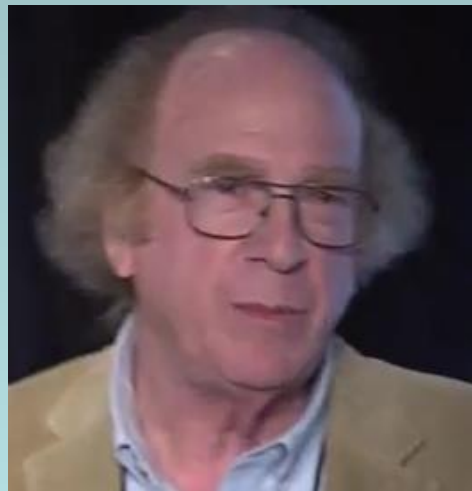








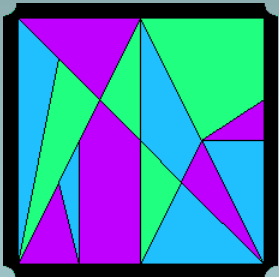
And somewhere in the middle are these humans on a planet blue —  
They have minds that play with puzzles, math and the magic they can do.



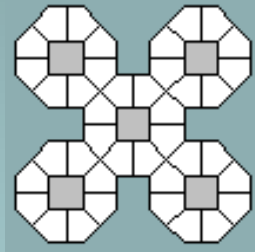
# 144

Wrapping up the last line, with a 12-times-12 word string — I counted them with care — Here is the list of all our games where you'll find pentominoes demand their share:

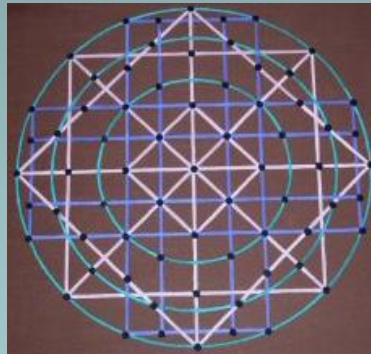
**Archimedes' Square**



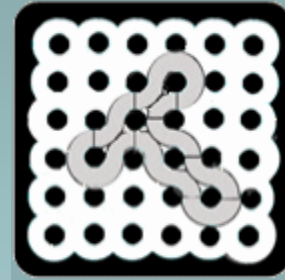
**Boats**



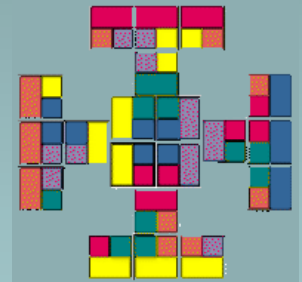
**Brace**



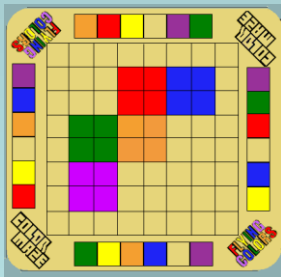
**ChooChooLoops**



**Color Up**



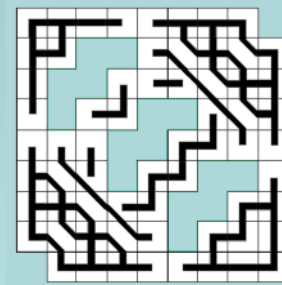
**Colormaze**



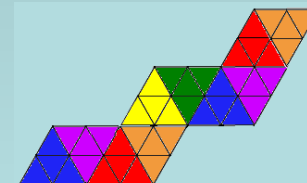
**Combinatorix**



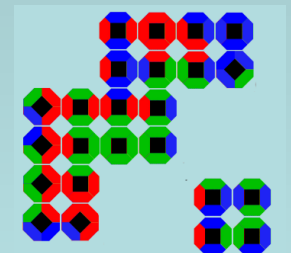
**Dezign-8**



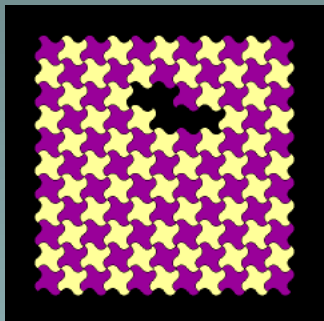
**Diamond Rainbow**



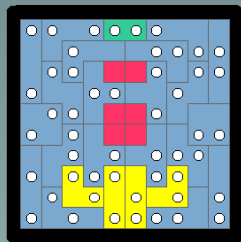
**Doris**



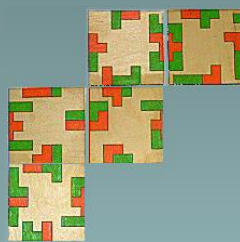
Dual Quintachex



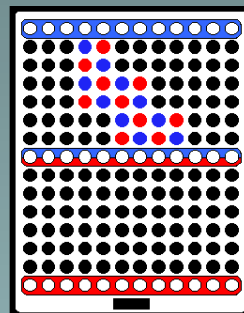
Fill-Agree



Fractured Fives



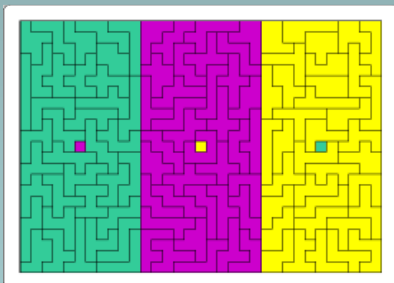
Gallop



Grand Multimatch



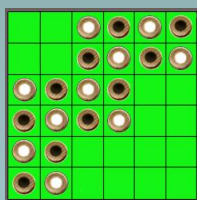
Heptominoes



Hexacube



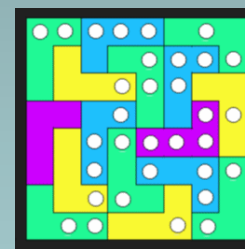
Leap



Lemma



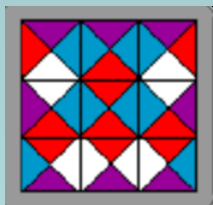
L-Sixteen



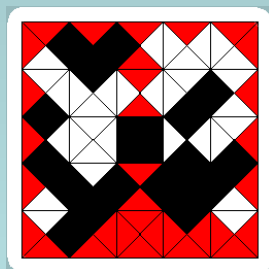
Mini-lamond  
Ring



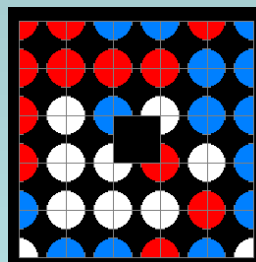
MiniMatch I



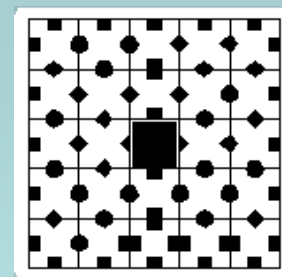
Multimatch I



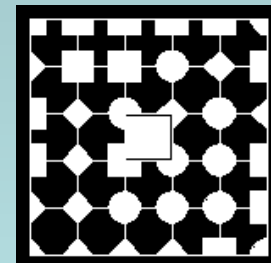
Multimatch II



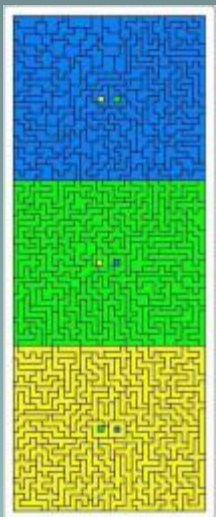
Multitouch I



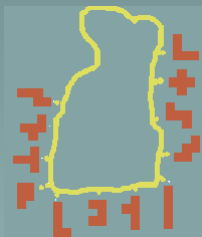
Multitouch II



Octominoes



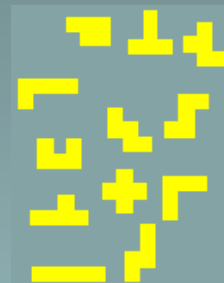
Pentomino necklace



Perplexing Pyramid



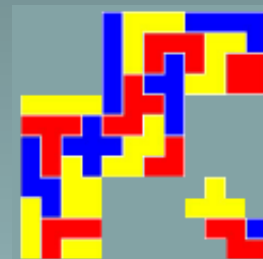
Pocket Pentominoes



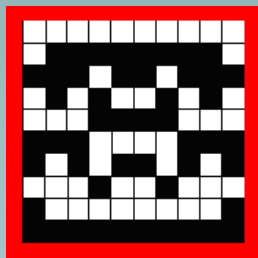
Pocket Vees



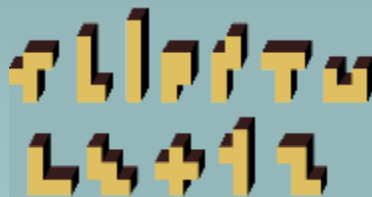
Poly-5



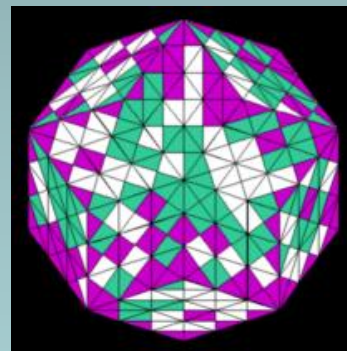
Quintapaths



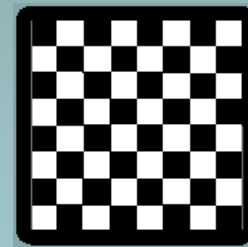
Quintillions



Rhom-Antics



Quintachex



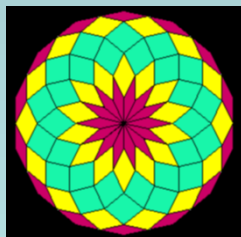
Quantum



Rhombiominos



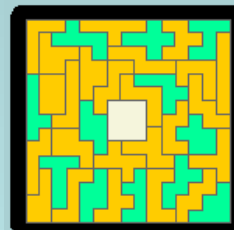
RhombStar-7



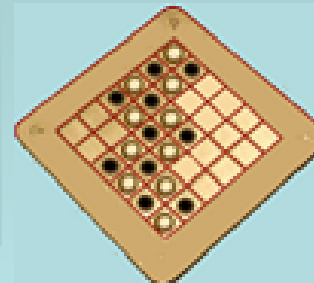
Rombix Jr.



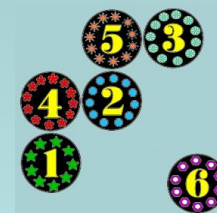
Sextillions



Six by Six



Six Disks





**Snowflake  
Square**



**Snowflake  
Super Square**



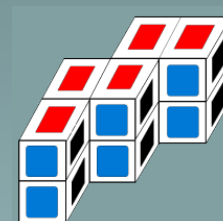
**Super  
Quintillions**



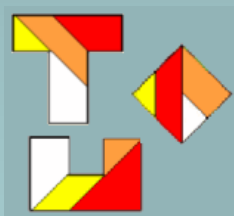
**Ten-Yen**



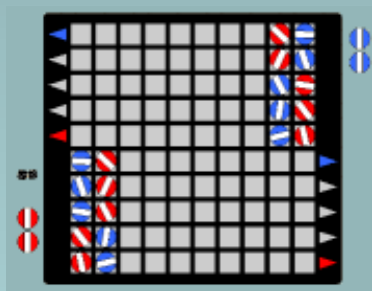
**Throw a Fit**



**Tiny Tans**



**Transpose**



**Triangoes Jr.**



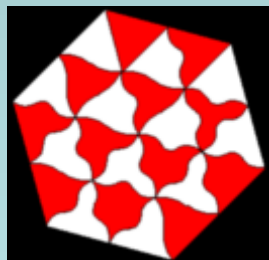
**Triangoes**



**Triangle-8**



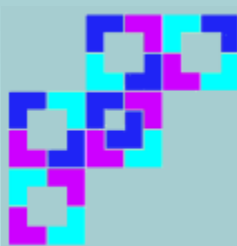
**Trifolia**



**Trio in a Tray**



**Vee-21**



**Void**



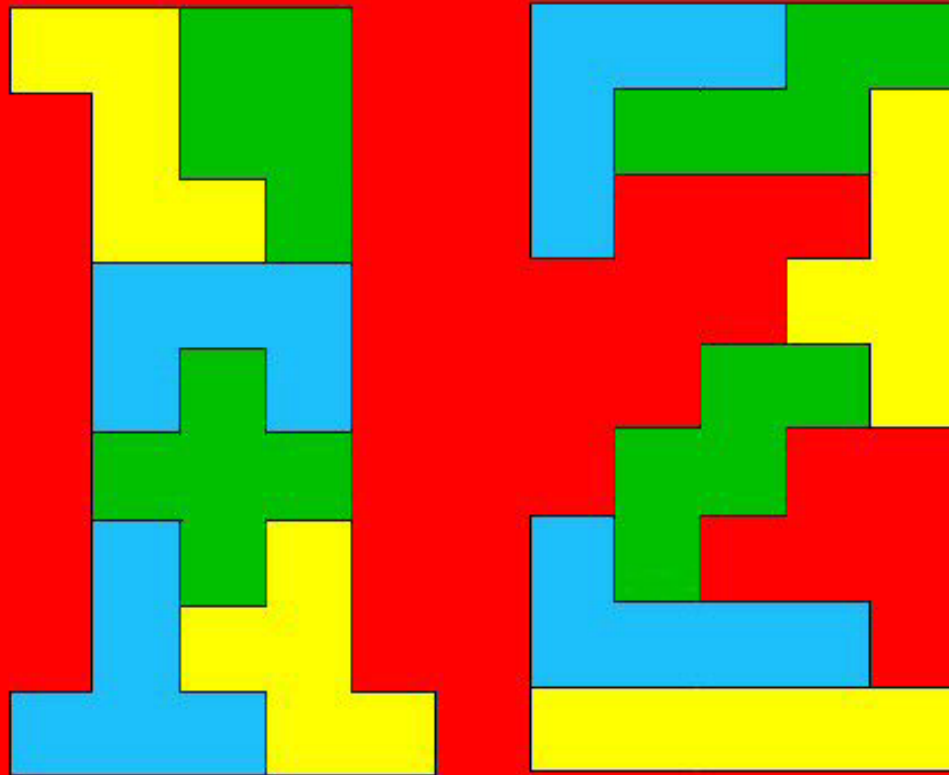
**and Warp-30**



**So thank you, Sol, for what you started,  
And thank you, Martin, for what you imparted,  
And thank you, World, for what you hearted.**



## A pentomino Fib



Never-ending